MEDAN ELEKTROMAGNETIK RELATIVISTIK BINTANG NEUTRON STASIONER

SKRIPSI

Untuk memenuhi sebagian syarat memperoleh
Derajat Sarjana S-1 Program Studi Fisika

Moh. Alifaki
07620008

PROGRAM STUDI FISIKA
FAKULTAS SAINS DAN TEKNOLOGI
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Universitas Islam Negeri Sunan Kalijaga FM-UINSK-BM-05-07/R0

PENGESAHAN SKRIPSI/TUGAS AKHIR

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Yang dipersiapkan dan disusun oleh :
Nama : Moh. Alifaki
NIM : 07620008
Telah dimunaqasyahkan pada : 18 Agustus 2015
Nilai Munaqasyah : A
Dan dinyatakan telah diterima oleh Fakultas Sains dan Teknologi UIN Sunan Kalijaga

TIM MUNAQASYAH :

Ketua Sidang

Dr.rer.nat. M. Farchani Rosyid
NIP.19680717199431000

Penguji I

Atsnaita Yasrina, M.Sc.

Penguji II

Joko Purwanto, M.Sc
NIP. 19820306 200912 1 002

Yogyakarta, 21 Agustus 2015
UIN Sunan Kalijaga
Fakultas Sains dan Teknologi
Dekan

KEMENTERIAN AGAMA
FAKULTAS SAINS DAN TEKNOLOGI
YOGYAKARTA
UIN SUNAN KALIJAGA

Dr. Hj. Maizer Said Nahdi, M.Si
NIP. 19550427 198403 2 001

SURAT PERNYATAAN KEASLIAN SKRIPSI

Yang bertanda tangan di bawah ini:

Nama	: Moh. Alifaki
NIM	: 07620008
Prodi/Smt	: Fisika / XVI
Fakultas	: Sains dan Teknologi

Dengan ini saya menyatakan bahwa skripsi ini tidak terdapat karya yang pernah diajukan untuk memperoleh gelar kesarjanaan di suatu Perguruan Tinggi, dan sepanjang pengetahuan saya juga tidak terdapat karya atau pendapat yang pernah ditulis atau diterbitkan orang lain, kecuali yang secara tertulis diacu dalam naskah ini dan disebutkan dalam daftar pustaka.

Yogyakarta, 08 Juli 2015

Yang Menyatakan,

METERAI TEMPEL
9D045ADF340024469
6000 ENAM RIBU RUPIAH

Moh. Alifaki
NIM. 07620008

SURAT PERSETUJUAN SKRIPSI/TUGAS AKHIR

Hal : Pengajuan Munaqosyah
Lamp :

Kepada
Yth. Dekan Fakultas Sains dan Teknologi
UIN Sunan Kalijaga Yogyakarta
di Yogyakarta

Assalamu'alaikum wr. wb.

Setelah membaca, meneliti, memberikan petunjuk dan mengoreksi serta mengadakan perbaikan seperlunya, maka kami selaku pembimbing berpendapat bahwa skripsi Saudara:

Nama : M Ali Fakih
NIM : 07620008
Judul Skripsi : Medan Elektromagnetik Relativistik Bintang Neutron Stasioner

sudah dapat diajukan kembali kepada Program Studi Fisika Fakultas Sains dan Teknologi UIN Sunan Kalijaga Yogyakarta sebagai salah satu syarat untuk memperoleh gelar Sarjana Strata Satu dalam Program Studi Fisika

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Wassalamu'alaikum wr. wb.

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Pembimbing I

Dr. rer. nat. Muhammad Farchani Rosyid
NIP. 19680717199431005

Pembimbing II

a.n.

Atsnaita Yasrina, M. Sc

Skripsi ini saya persembahkan kepada keluarga besar:

Embu' Maryati
Ema' Rusipa
Almarhum *Eppa'* Asyikurrahman
Ale' Ahmad Sobiri Yanto

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Penulis

MEDAN ELEKTROMAGNETIK RELATIVISTIK BINTANG NEUTRON STASIONER

Moh. Alifaki
07620008

Intisari

Telah dirumuskan persamaan medan elektromagnetik relativistik untuk bintang neutron stasioner dengan batasan konduktivitas di dalam bintang berhingga dan seragam serta medan magnet yang dikaji hanya medan magnet internal bintang. Hasilnya adalah tiga komponen persamaan medan elektromagnetik relativistik bintang neutron stasioner, yaitu komponen radial, komponen polar dan komponen toroidal.

Kata-kata kunci: Bintang neutron, medan elektromagnetik relativistik.

RELATIVISTIC ELECTROMAGNETIC FIELD OF THE STATIONARY NEUTRON STAR

Moh. Alifaki
07620008

Abstract

Equation of relativistic electromagnetic field toward stationary neutron star has been formulated with constraint conductivity within finite and uniform star's interior and the studied magnetic field is only internal magnetic field of the star. The result which is identified for component equation of relativistic electromagnetic field in stationary neutron star has revealed three findings: redial component, polar component and toroidal component.

Keywords: Neutron star, relativistic electromagnetic field.

DAFTAR ISI

BAB I
PENDAHULUAN

1.1. Latar Belakang Masalah

Bintang neutron merupakan kelompok bintang kompak di alam semesta. Kekompakan bintang neutron tersebut bisa dilihat dari besar massa dan rapat massanya. Jika dibandingkan dengan Matahari, masa bintang neutron adalah $M \sim$ 1-2 $M_{\odot}$ ($M_{\odot}$ = 2 × 10^{33} kg) dengan radius hanya $R \approx$ 10-14 km. Adapun rapat massanya tidaklah seragam. Di luar intinya, kerapatan bintang neutron tiga kali lebih rapat dibanding inti atom (ρ_0 = 2.8 x 10^{14} $g\ cm^{-3}$), yaitu $\rho \sim 10^{15}$ $g\ cm^{-3}$. Sedangkan rapat inti bintang neutron besarnya lebih dari ρ_0 (Potekhin, 2011).

Oleh karena kekompakan bintang neutron itulah maka, untuk mengkajinya secara realistis, TRU (Teori Relativitas Umum) harus digunakan. Kekompakan sebuah objek relativistik ditentukan oleh parameter $x_g = r_g/R$, dengan $r_g = 2GM/c^2$ = 2.95 $M/M_{\odot}$ disebut radius Schwarzschild (Potekhin, 2011). Misalnya untuk bintang neutron *kanonik* yang punya M = 1.4 $M_{\odot}$ dan R = 12 km, maka r_g = 4.13 km dan $x_g = 0.34$. Hasil ini sesuai dengan kenyataan bahwa parameter kekompakan bintang secara umum $x_g \ll 1$. Artinya, jika x_g = 1 adalah adalah 100%, maka nilai parameter 0.34 menunjukkan bintang neutron *kanonik* punya efek relativistik 10%.

Sementara itu, bintang neutron dikenal memiliki medan magnetik yang paling kuat di alam semesta (Potekhin, 2011). Besar medan magnet di permukaannya sekitar $B{\sim}10^8 - 10^{13}G$, sedangkan di bagian dalamnya mencapai $B{\sim}10^{15}G$ (Bhattacharya, 2002). Umur medan magnet bintang neutron $\pm 15 \times 10^9\ tahun$, lebih lama dibanding umur alam semesta $\pm 13 \times 10^9$ tahun (Potekhin, 2011). Namun anomali tersebut segera dapat teratasi, karena medan magnet bintang neutron menyusut secara dramatis, yaitu dari $B{\sim}10^{12}\ G$ menjadi $B{\sim}10^8\ G$ dalam skala waktu 5×10^6 tahun (Zhang, 1997). Diduga bahwa penyusutan medan magnet ini disebabkan oleh tiga hal, yaitu ekspulsi spindown-induksi fluks, evolusi ohmik pada kerak dan peristiwa akresi atau "memakannya" bintang neutron terhadap materi-materi di sekitar permukaannya atau terhadap bintang kawanannya untuk bintang neutron sistem ganda (Bhattacharya, 2002).

Secara logis, memang tidak mungkin elemen alam semesta seperti medan magnet bintang neutron berumur lebih lama dibanding alam semesta itu sendiri. Seluruh elemen alam semesta sudah ditetapkan ukurannya masing-masing oleh Allah Subhanahu Wa Ta'ala, dengan ukuran yang tidak mungkin menyalahi akal sehat manusia. Sebelum manusia menyadari hal tersebut, jauh-jauh hari Allah Subhanahu Wa Ta'ala sesungguhnya sudah mengisyaratkannya dalam al-Qur'an surah al-Qomar [54] ayat 49 (Depag RI, 2007):

"*Sesungguhnya Kami menciptakan segala sesuatu dengan ukuran*".

Dan di dalam al-Qur'an surah al-Jaatsiyah [45] ayat 13 (Depag RI, 2007), Allah Subhanahu Wa Ta'ala berfirman:

"*Dan Dia menundukkan untukmu apa yang ada di langit dan apa yang ada di bumi semuanya (sebagai rahmat) daripadaNya. Sesungguhnya pada yang demikian itu benar-benar terdapat tanda-tanda (kekuasaan Allah) bagi kaum yang berpikir*".

Dalam (QS 54: 49), Allah Subhanahu Wa Ta'ala menegaskan bahwa tidak ada satu pun elemen alam semesta yang tidak terukur. Hanya saja Allah Subhanahu Wa Ta'ala membebaskan kepada manusia berpikir tentang "alat ukur" macam apa yang dapat dipakai untuk mengukur ukuran elemen alam semesta yang telah ditetapkan oleh Allah Subhanahu Wa Ta'ala tersebut. Alat ukur yang manusia pakai untuk mengukur elemen alam semesta, dalam konteks ini bintang neutron, adalah teori-teori fisika yang sudah difalsafikasi (sudah memenuhi standar-standar logis konvensional).

Sementara kata "*Dia (Allah) menundukkan*" dalam (QS 45: 13) berarti bahwa alat ukur yang dipakai manusia, sepanjang memenuhi terma-terma logis, "ukuran" alam semesta tidak akan pernah bertentangan dengan akal sehat manusia, karena dengan kasih-sayangNya, pada mulanya Allah Subhanahu Wa Ta'ala telah "menundukkan" alam semesta ini kepada manusia. Kalau manusia tidak mampu memahami anomali dari "ukuran" alam semesta bukan berarti Allah Subhanahu Wa Ta'ala menyembunyikannya (tidak "menundukkan"), tetapi pasti alat ukur yang digunakan oleh manusia tersebut bermasalah.

Sekarang sudah ditemukan sumber pemecah anomali dari umur medan magnet bintang neutron tersebut, yakni bahwa medan magnet bintang neutron menyusut. Dengan adanya gejala penyusutan medan magnet bintang neutron yang ditengarai dikarenakan peristiwa akresi bintang neutron terhadap bintang kawanannya menjadi alasan logis tentang bahwa umur medan magnet bintang neutron tidak mungkin melebihi umur alam semesta. Di sini berarti alat ukur fisis yang digunakan untuk menelaah medan magnet bintang neutron sudah benar, karena dengan alat ukur tersebut fenomena medan magnet bintang neutron menjadi logis.

Berangkat dari kenyataan tersebut, cara untuk mengetahui penyusutan medan magnet bintang neutron, diperlukan persamaan dinamika medan magnet pada bintang neutron itu sendiri. Dalam rangka memperoleh persamaan dinamika medan magnetik bintang neutron, para peneliti sebelumnya menggunakan beberapa model dan batasan. Ada yang membatasi medan magnetik bintang neutron dalam dua dimensi (bintang dianggap flat), ada yang membatasi untuk bintang neutron non-relativistik stasioner, ada yang membatasi untuk bintang neutron relativistik berotasi lambat, dan ada juga yang membatasi untuk bintang neutron non-relativistik berotasi cepat.

Di dalam penelitian ini, akan dikaji dinamika medan magnetik (yang digambarkan oleh persamaan Maxwell) bintang neutron yang stasioner dalam kerangka TRU. Ini dimaksudkan untuk memperoleh gambaran tentang bagaimanakah dinamika medan elektromagnetik pada bintang neutron stasioner. Hasilnya diharapkan dapat membantu untuk merumuskan persamaan penyusutan medan elektromagnetik relativistik bintang neutron stasioner pada penelitian-penelitian berikutnya di masa mendatang.

Tidak hanya itu, hasil rumusan yang diperoleh dalam penelitian ini nantinya diharapkan dapat dipergunakan sebagai bahan kajian pengembangan perumusan dinamika medan elektromagnetik relativistik pada bintang neutron non-stasioner, baik berotasi lambat maupun berotasi cepat. Kajian tersebut diharapkan mengandaikan bagian luar bintang terdapat materi sedemikian sehingga memungkinkan adanya proses akresi. Hal ini dibutuhkan karena peristiwa akresi diduga menjadi penyebab utama menyusutnya medan magnet di bintang neutron.

1.2. Rumusan Masalah

Berdasarkan uraian latar belakang masalah di atas, maka permasalahan yang akan diteliti dalam penelitian ini dirumuskan sebagai berikut:

Bagaimana rumusan persamaan dinamika medan magnet relativistik bintang neutron stasioner?

1.3. Tujuan Penelitian

Berdasarkan rumusan masalah tersebut, maka penelitian ini memiliki tujuan, yaitu untuk merumuskan persamaan dinamika medan elektromagnetik relativistik bintang neutron stasioner.

1.4. Batasan Penelitian

Untuk mencapai tujuan yang diinginkan, maka batasan penelitian ini ditentukan sebagai berikut:

1. Konduktivitas listriknya berhingga dan di dalam bintang bernilai seragam.
2. Medan magnet yang dikaji hanyalah medan magnet internal bintang neutron.
3. Bintang neutron dalam keadaan stasioner.
4. Bintang neutron tidak mengakresi.

1.5. Manfaat Penelitian

Setelah penelitian ini sudah dilakukan, maka hasil yang diperoleh diharapkan dapat memberikan manfaat, antara lain:

1. Memberikan pemahaman yang lebih luas khususnya kepada peneliti sendiri tentang teori medan elektromagnetik relativistik dalam penerapannya terhadap bintang neutron stasioner.
2. Dapat dipergunakan sebagai pedoman untuk memformulasikan persamaan penyusutan medan magnet bintang neutron yang selama ini menjadi isu krusial dalam astrofisika.
3. Dapat dipergunakan sebagai bahan kajian pengembangan perumusan dinamika medan elektromagnetik bintang neutron relativistik non-stasioner, baik yang berotasi lambat maupun berotasi cepat, serta menyusutnya medan magnet bintang neutron yang diduga diakibatkan oleh aktivitas akresi bintang.

BAB V
KESIMPULAN DAN SARAN

5.1. Kesimpulan

Berdasarkan pada tujuan penelitian ini, maka rumusan dinamika medan elektromagnetik relativistik bintang neutron stasioner disajikan oleh persamaan (4.94) untuk komponen radial, persamaan (4.95) untuk komponen polar, dan persamaan (4.96) untuk komponen toroidal. Penelitian ini hanya menghasilkan persamaan dinamika medan elektromagnetik pada bintang neutron stasioner. Oleh karenanya persamaan ini tidak dapat digunakan untuk menghitung penyusutan medan magnet bintang neutron. Akan tetapi persamaan ini sangat penting, karena persamaan ini dapat membantu para peneliti lain di masa mendatang untuk memformulasikan persamaan penyusutan medan magnet pada bintang neutron yang selama ini menjadi isu krusial dalam astrofisika.

5.2. Saran

Kajian penelitian ini masih dibatasi pada tiga ruang lingkup, yaitu medan magnet internal bintang neutron, konduktivitas listrik di dalam bintang bernilai seragam, dan bintang neutron dianggap stasioner. Oleh karena itu, maka peneliti merekomendasikan beberapa saran untuk penelitian-penelitian berikutnya, yaitu:

1. Merumuskan persamaan penyusutan medan magnet bintang neutron stasioner berdasarkan pada hasil persamaan dinamika medan elektromagnetik relativistik bintang neutron stasioner di penelitian ini.
2. Merumuskan persamaan dinamika medan elektromagnetik relativistik pada bintang neutron stasioner dengan menganggap konduktivitas listriknya di dalam bintang tidak seragam.
3. Merumuskan persamaan dinamika medan elektromagnetik relativistik pada bintang neutron stasioner dengan menggunakan ruang-waktu eksternal bintang.

4. Merumuskan persamaan dinamika medan elektromagnetik relativistik pada bintang neutron stasioner bila dianggap ada perpindahan arus dari luar bintang ke dalam bintang.
5. Merumuskan persamaan dinamika medan elektromagnetik relativistik pada bintang neutron non-stasioner, baik berotasi lambat maupun berotasi cepat, baik adanya perpindahan arus atau tidak.
6. Merumuskan persamaan dinamika medan elektromagnetik baik pada bintang neutron stasioner maupun non-stasioner menggunakan ruang-waktu bermetrik elipsoid, mengingat pada kenyataannya kontur bintang bukanlah simetri bola, akan tetapi elipsoidal.

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LAMPIRAN A
PENURUNAN PERSAMAAN MAXWELL I

Geometri bintang neutron stasioner diberikan oleh

$$ds^2 = -e^{2\phi(r)}dt^2 + e^{2\Lambda(\mathrm{r})}dr^2 + r^2 d\theta^2 + r^2 sin^2\theta\, d\varphi^2 \tag{A.1}$$

dengan

$$g_{\alpha\beta} = \begin{bmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{bmatrix} = \begin{bmatrix} -e^{2\phi(r)} & 0 & 0 & 0 \\ 0 & e^{2\Lambda(\mathrm{r})} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 sin^2\theta \end{bmatrix} \tag{A.2}$$

dan nilai tiap komponen tak-nolnya

$$\begin{gathered} g_{00} = -e^{2\phi(r)} \\ g_{11} = e^{2\Lambda(\mathrm{r})} \\ g_{22} = r^2 \\ g_{33} = r^2 sin^2\theta. \end{gathered} \tag{A.3}$$

Dengan demikian, metrik skalarnya

$$g = det\left|g_{\alpha\beta}\right| = -e^{2\phi(r)}\begin{bmatrix} e^{2\Lambda(\mathrm{r})} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2 sin^2\theta \end{bmatrix} = -e^{2\phi(r)}\left\{e^{2\Lambda(\mathrm{r})} r^4 sin^2\theta\right\}$$
$$= -e^{2[\phi+\Lambda](r)} r^4 sin^2\theta$$

(A.4)

dan

$$\sqrt{-g} = \sqrt{e^{2(\phi+\Lambda)(r)} r^4 sin^2\theta} = e^{[\phi+\Lambda](r)} r^2 sin\,\theta$$

(A.5)

Dari kenyataan bahwa

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

maka diketahui kecepatan medan kovarian dan kontravarian

$$u_0 = e^{\phi(r)}(-1,0,0,0)$$
$$u^0 = e^{-\phi(r)}(1,0,0,0).$$

(A.6)

Sementara itu, Persamaan Maxwell I seperti ditunjukkan oleh persamaan (2.131). Dari persamaan tersebut, dibutuhkan perhitungan tensor kuat-medan elektromagnetik yang persamaannya diberikan oleh (2.141)

$$F_{\alpha\beta} = 2u_{[\alpha}E_{\beta]} + \eta_{\alpha\beta\gamma\delta}u^\gamma B^\delta = u_\alpha E_\beta - u_\beta E_\alpha + \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta}u^\gamma B^\delta$$
$$F_{\beta\alpha} = 2u_{[\beta}E_{\alpha]} + \eta_{\beta\alpha\gamma\delta}u^\gamma B^\delta = u_\beta E_\alpha - u_\alpha E_\beta + \sqrt{-g}\epsilon_{\beta\alpha\gamma\delta}u^\gamma B^\delta$$
$$= -\left(u_\alpha E_\beta - u_\beta E_\alpha + \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta}u^\gamma B^\delta\right) = -F_{\alpha\beta}$$

(A.7)

bersifat anti-simetrik. Berikut komponen-komponennya:

3. Untuk $\alpha = \beta$

$$F_{\alpha\alpha} = u_\alpha E_\alpha - u_\alpha E_\alpha + \sqrt{-g}\epsilon_{\alpha\alpha\gamma\delta}u^\gamma B^\delta = \sqrt{-g}\epsilon_{\alpha\alpha\gamma\delta}u^\gamma B^\delta.$$

Karena simbol Levi-Civita untuk $\epsilon_{\alpha\alpha\gamma\delta} = 0$, maka

$$F_{00} = F_{11} = F_{22} = F_{33} = 0 \tag{A.8}$$

4. Untuk $\alpha \neq \beta$

$$F_{\alpha\beta} = 2u_{[\alpha}E_{\beta]} + \eta_{\alpha\beta\gamma\delta}u^\gamma B^\delta = u_\alpha E_\beta - u_\beta E_\alpha + \sqrt{-g}\epsilon_{\alpha\beta\gamma\delta}u^\gamma B^\delta$$

- Untuk $\alpha = 0$ dan $\beta = 1$

$$F_{01} = u_0 E_1 - u_1 E_0 + \sqrt{-g}(\epsilon_{0123}u^2 B^3 - \epsilon_{0132}u^3 B^2)$$

Karena $u_1 = 0$, $u^2 = 0$ dan $u^3 = 0$, maka

$$F_{01} = u_0 E_1 = -e^{\phi(r)}E_r = -F_{10} \tag{A.9}$$

- Untuk $\alpha = 0$ dan $\beta = 2$

$$F_{02} = u_0E_2 - u_2E_0 + \sqrt{-g}(\epsilon_{0231}u^3B^1 - \epsilon_{0213}u^1B^3)$$

$$F_{02} = u_0E_2 = -e^{\phi(r)}E_\theta = -F_{20} \tag{A.10}$$

- Untuk $\alpha = 0$ dan $\beta = 3$

$$F_{03} = u_0E_3 - u_3E_0 + \sqrt{-g}(\epsilon_{0312}u^1B^2 - \epsilon_{0321}u^2B^1)$$

$$F_{03} = u_0E_3 = -e^{\phi(r)}E_\varphi = -F_{30} \tag{A.11}$$

- Untuk $\alpha = 1$ dan $\beta = 2$

$$F_{12} = u_1E_2 - u_2E_1 + \sqrt{-g}(\epsilon_{1230}u^3B^0 - \epsilon_{1203}u^0B^3)$$

$$F_{12} = \sqrt{-g}(\epsilon_{1203}u^0B^3)$$

Dengan permutasi simbol Levi-Civita $\epsilon_{1203} = -\epsilon_{0213} = -(-\epsilon_{0123}) = +1$, maka

$$F_{12} = e^{[\phi+\Lambda](r)}r^2 sin\,\theta e^{-\phi(r)}B^\varphi = e^{\Lambda(r)}r^2 sin\,\theta\, B^\varphi = -F_{21} \tag{A.12}$$

- Untuk $\alpha = 1$ dan $\beta = 3$

$$F_{13} = u_1E_3 - u_3E_1 + \sqrt{-g}(\epsilon_{1302}u^0B^2 - \epsilon_{1320}u^2B^0)$$

$$F_{13} = \sqrt{-g}(\epsilon_{1302}u^0B^2)$$

Dengan permutasi simbol Levi-Civita $\epsilon_{1302} = -\epsilon_{0312} = -(-\epsilon_{0132}) = -\epsilon_{0123} = -1$, maka

$$F_{13} = -e^{[\phi+\Lambda](r)}r^2 sin\ \theta e^{-\phi(r)}B^\theta = -e^{\Lambda(r)}r^2 sin\ \theta\ B^\theta = -F_{31} \tag{A.13}$$

- Untuk $\alpha = 2$ dan $\beta = 3$

$$F_{23} = u_2E_3 - u_3E_2 + \sqrt{-g}(\epsilon_{2301}u^0B^1 - \epsilon_{2310}u^1B^0)$$

$$F_{23} = \sqrt{-g}(\epsilon_{2301}u^0B^2)$$

Dengan permutasi simbol Levi-Civita $\epsilon_{2301} = -\epsilon_{0321} = -(-\epsilon_{0123}) = +1$, maka

$$F_{23} = e^{[\phi+\Lambda](r)}r^2 sin\ \theta e^{-\phi(r)}B^r = e^{\Lambda(r)}r^2 sin\ \theta\ B^r = -F_{32} \tag{A.14}$$

Jadi, tensor kuat-medan elektromagnetik kovarian diberikan oleh

$$F_{\alpha\beta} = \begin{bmatrix} 0 & -e^{\phi(r)}E_r & -e^{\phi(r)}E_\theta & -e^{\phi(r)}E_\varphi \\ e^{\phi(r)}E_r & 0 & e^{\Lambda(r)}r^2 sin\ \theta\ B^\varphi & -e^{\Lambda(r)}r^2 sin\ \theta\ B^\theta \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\ \theta\ B^\varphi & 0 & e^{\Lambda(r)}r^2 sin\ \theta\ B^r \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\ \theta\ B^\theta & -e^{\Lambda(r)}r^2 sin\ \theta\ B^r & 0 \end{bmatrix} \tag{A.15}$$

Turunan parsial tensor kuat-medan elektromagnetik adalah

5. Untuk $\alpha = \alpha = \alpha$

   Karena $F_{\alpha\alpha\alpha} = 0$, maka

$$F_{00,0} = F_{11,1} = F_{22,2} = F_{33,3} = 0 \tag{A.16}$$

6. Untuk $\alpha = \beta \neq \gamma$

   Karena $F_{\alpha\alpha} = 0$ yang mengimplikasikan $F_{\alpha\alpha,\gamma} = 0$, maka

$$\begin{gathered} F_{00,1} = F_{00,2} = F_{00,3} = 0 \\ F_{11,0} = F_{11,2} = F_{11,3} = 0 \\ F_{22,0} = F_{22,1} = F_{00,3} = 0 \\ F_{33,0} = F_{33,1} = F_{33,2} = 0 \end{gathered} \tag{A.17}$$

7. Untuk $\alpha = \gamma \neq \beta$

   - Untuk $\alpha = \gamma = 0$

     Dengan $\beta = 1$

$$F_{01,0} = \frac{\partial(F_{01})}{\partial t} = \frac{\partial\left(-e^{\phi(r)}E_r\right)}{\partial t} = -\frac{\partial e^{\phi(r)}}{\partial t}E_r - \frac{\partial E_r}{\partial t}e^{\phi(r)} = -\frac{\partial E_r}{\partial t}e^{\phi(r)} = -F_{10,0} \tag{A.18}$$

     Dengan $\beta = 2$

$$F_{02,0} = \frac{\partial(F_{02})}{\partial t} = \frac{\partial\left(-e^{\phi(r)}E_\theta\right)}{\partial t} = -\frac{\partial e^{\phi(r)}}{\partial t}E_\theta - \frac{\partial E_\theta}{\partial t}e^{\phi(r)} = -\frac{\partial E_\theta}{\partial t}e^{\phi(r)} = -F_{20,0}$$

(A.19)

Dengan $\beta = 3$

$$F_{03,0} = \frac{\partial(F_{03})}{\partial t} = \frac{\partial\left(-e^{\phi(r)}E_\varphi\right)}{\partial t} = -\frac{\partial e^{\phi(r)}}{\partial t}E_\varphi - \frac{\partial E_\varphi}{\partial t}e^{\phi(r)} = -\frac{\partial E_\varphi}{\partial t}e^{\phi(r)}$$
$$= -F_{30,0}$$

(A.20)

- Untuk $\alpha = \gamma = 1$

Dengan $\beta = 0$

$$F_{10,1} = \frac{\partial(F_{10})}{\partial r} = \frac{\partial\left(e^{\phi(r)}E_r\right)}{\partial r} = \left(e^{\phi(r)}E_r\right)_{,r} = -F_{01,1}$$

(A.21)

Dengan $\beta = 2$

$$F_{12,1} = \frac{\partial(F_{12})}{\partial r} = \frac{\partial\left(e^{\Lambda(r)}r^2 sin\,\theta\, B^\varphi\right)}{\partial r} = sin\,\theta\left(e^{\Lambda(r)}r^2B^\varphi\right)_{,r} = -F_{21,1}$$

(A.22)

Dengan $\beta = 3$

$$F_{13,1} = \frac{\partial(F_{13})}{\partial r} = \frac{\partial\left(-e^{\Lambda(r)}r^2 sin\,\theta\, B^\theta\right)}{\partial r} = -sin\,\theta\left(e^{\Lambda(r)}r^2B^\theta\right)_{,r} = -F_{31,1}$$

(A.23)

- Untuk $\alpha = \gamma = 2$

  Dengan $\beta = 0$

$$F_{20,2} = \frac{\partial(F_{20})}{\partial\theta} = \frac{\partial\left(e^{\phi(r)}E_\theta\right)}{\partial\theta} = e^{\phi(r)}(E_\theta)_{,\theta} = -F_{02,2}$$

(A.24)

Dengan $\beta = 1$

$$F_{21,2} = \frac{\partial(F_{21})}{\partial\theta} = \frac{\partial\left(-e^{\Lambda(r)}r^2 sin\,\theta\, B^\varphi\right)}{\partial\theta} = -e^{\Lambda(r)}r^2(sin\,\theta\, B^\varphi)_{,\theta} = -F_{12,2}$$

(A.25)

Dengan $\beta = 3$

$$F_{23,2} = \frac{\partial(F_{23})}{\partial\theta} = \frac{\partial\left(e^{\Lambda(r)}r^2 sin\,\theta\, B^r\right)}{\partial\theta} = e^{\Lambda(r)}r^2(sin\,\theta\, B^r)_{,\theta} = -F_{32,3}$$

(A.26)

- Untuk $\alpha = \gamma = 3$

  Dengan $\beta = 0$

$$F_{30,3} = \frac{\partial(F_{30})}{\partial\varphi} = \frac{\partial\left(e^{\phi(r)}E_\varphi\right)}{\partial\varphi} = e^{\phi(r)}\left(E_\varphi\right)_{,\varphi} = -F_{03,3}$$

(A.27)

Dengan $\beta = 1$

$$F_{31,3} = \frac{\partial(F_{31})}{\partial\varphi} = \frac{\partial\left(e^{\Lambda(r)}r^2 sin\,\theta\; B^\theta\right)}{\partial\varphi} = e^{\phi(r)}r^2 sin\,\theta\,\left(B^\theta\right)_{,\varphi} = -F_{13,3}$$

(A.28)

Dengan $\beta = 2$

$$F_{32,3} = \frac{\partial(F_{32})}{\partial\varphi} = \frac{\partial\left(-e^{\Lambda(r)}r^2 sin\,\theta\; B^r\right)}{\partial\varphi} = -e^{\phi(r)}r^2 sin\,\theta\,(B^r)_{,\varphi} = -F_{23,3}$$

(A.29)

8. Untuk $\alpha \neq \gamma \neq \beta$

   $\alpha = 0$ dan $\beta = 1$

   - Dengan $\gamma = 2$

$$F_{01,2} = \frac{\partial(F_{01})}{\partial\theta} = \frac{\partial\left(-e^{\phi(r)}E_r\right)}{\partial\theta} = -e^{\phi(r)}(E_r)_{,\theta} = -F_{10,2}$$

(A.30)

   - Dengan $\gamma = 3$

$$F_{01,3} = \frac{\partial(F_{01})}{\partial\varphi} = \frac{\partial\left(-e^{\phi(r)}E_r\right)}{\partial\varphi} = -e^{\phi(r)}(E_r)_{,\varphi} = -F_{10,3}$$

(A.31)

   $\alpha = 0$ dan $\beta = 2$

   - Dengan $\gamma = 1$

$$F_{02,1} = \frac{\partial(F_{02})}{\partial r} = \frac{\partial\left(-e^{\phi(r)}E_\theta\right)}{\partial r} = -\left(e^{\phi(r)}E_\theta\right)_{,r} = -F_{20,1}$$

(A.32)

- Dengan $\gamma = 3$

$$F_{02,3} = \frac{\partial(F_{02})}{\partial\varphi} = \frac{\partial\left(-e^{\phi(r)}E_\theta\right)}{\partial\varphi} = -e^{\phi(r)}(E_\theta)_{,\varphi} = -F_{20,3}$$

(A.33)

$\alpha = 0$ dan $\beta = 3$

- Dengan $\gamma = 1$

$$F_{03,1} = \frac{\partial(F_{03})}{\partial r} = \frac{\partial\left(-e^{\phi(r)}E_\varphi\right)}{\partial r} = -\left(e^{\phi(r)}E_\varphi\right)_{,r} = -F_{30,1}$$

(A.34)

- Dengan $\gamma = 2$

$$F_{03,2} = \frac{\partial(F_{03})}{\partial\theta} = \frac{\partial\left(-e^{\phi(r)}E_\varphi\right)}{\partial\theta} = -e^{\phi(r)}\left(E_\varphi\right)_{,\theta} = -F_{30,2}$$

(A.35)

$\alpha = 1$ dan $\beta = 2$

- Dengan $\gamma = 0$

$$F_{12,0} = \frac{\partial(F_{12})}{\partial t} = \frac{\partial\left(e^{\Lambda(r)}r^2 sin\,\theta\; B^\varphi\right)}{\partial t} = e^{\Lambda(r)}r^2 sin\,\theta\;(B^\varphi)_{,t} = -F_{21,0}$$

(A.36)

- Dengan $\gamma = 3$

$$F_{12,3} = \frac{\partial(F_{12})}{\partial\varphi} = \frac{\partial\left(e^{\Lambda(r)}r^2 sin\,\theta\; B^{\varphi}\right)}{\partial\varphi} = e^{\Lambda(r)}r^2 sin\,\theta\;(B^{\varphi})_{,\varphi} = -F_{21,3}$$

(A.37)

$\alpha = 1$ dan $\beta = 3$

- Dengan $\gamma = 0$

$$F_{13,0} = \frac{\partial(F_{13})}{\partial t} = \frac{\partial\left(-e^{\Lambda(r)}r^2 sin\,\theta\; B^{\theta}\right)}{\partial t} = -e^{\Lambda(r)}r^2 sin\,\theta\;\left(B^{\theta}\right)_{,t} = -F_{31,0}$$

(A.38)

- Dengan $\gamma = 2$

$$F_{13,2} = \frac{\partial(F_{13})}{\partial\theta} = \frac{\partial\left(-e^{\Lambda(r)}r^2 sin\,\theta\; B^{\theta}\right)}{\partial\theta} = -e^{\Lambda(r)}r^2\left(sin\,\theta\; B^{\theta}\right)_{,\theta} = -F_{31,2}$$

(A.39)

$\alpha = 2$ dan $\beta = 3$

- Dengan $\gamma = 0$

$$F_{23,0} = \frac{\partial(F_{23})}{\partial t} = \frac{\partial\left(e^{\Lambda(r)}r^2 sin\,\theta\; B^{r}\right)}{\partial t} = e^{\Lambda(r)}r^2 sin\,\theta\;(B^{r})_{,t} = -F_{32,0}$$

(A.40)

- Dengan $\gamma = 1$

$$F_{23,1} = \frac{\partial(F_{23})}{\partial r} = \frac{\partial\left(e^{\Lambda(r)}r^2 sin\,\theta\; B^{r}\right)}{\partial r} = sin\,\theta\;\left(e^{\Lambda(r)}r^2 B^{r}\right)_{,r} = -F_{32,1}$$

(A.41)

Persamaan Maxwell I tidak lenyap jika $\alpha \neq \gamma \neq \beta$. Maka komponen-komponennya dapat dituliskan sebagai berikut

- Untuk permutasi 012

$$F_{01,2} + F_{20,1} + F_{12,0} = 0$$
$$-e^{\phi(r)}(E_r)_{,\theta} + \left(e^{\phi(r)}E_\theta\right)_{,r} + e^{\Lambda(r)}r^2 sin\,\theta\,(B^\varphi)_{,t} = 0$$
$$e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial B^\varphi}{\partial t} = \left(e^{\phi(r)}E_\theta\right)_{,r} - e^{\phi(r)}(E_r)_{,\theta}$$

(A.42)

- Untuk permutasi 013

$$F_{01,3} + F_{30,1} + F_{13,0} = 0$$
$$-e^{\phi(r)}(E_r)_{,\varphi} + \left(e^{\phi(r)}E_\varphi\right)_{,r} e^{\Lambda(r)}r^2 sin\,\theta\,\left(B^\theta\right)_{,t} = 0$$
$$e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial B^\theta}{\partial t} = \left(e^{\phi(r)}E_\varphi\right)_{,r} - e^{\phi(r)}(E_r)_{,\varphi}$$

(A.43)

- Untuk permutasi 023

$$F_{02,3} + F_{30,2} + F_{23,0} = 0$$
$$-e^{\phi(r)}(E_\theta)_{,\varphi} + e^{\phi(r)}\left(E_\varphi\right)_{,\theta} + e^{\Lambda(r)}r^2 sin\,\theta\,(B^r)_{,t} = 0$$
$$e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial B^r}{\partial t} = e^{\phi(r)}\left(E_\varphi\right)_{,\theta} - e^{\phi(r)}(E_\theta)_{,\varphi}$$

(A.44)

- Untuk permutasi 123

$$F_{12,3} + F_{31,2} + F_{23,1} = 0$$

$$e^{\Lambda(r)} r^2 sin\,\theta\, (B^{\varphi})_{,\varphi} + e^{\Lambda(r)} r^2 \left(sin\,\theta\, B^{\theta}\right)_{,\theta} + sin\,\theta\, \left(e^{\Lambda(r)} r^2 B^{r}\right)_{,r} = 0$$

(A.45)

LAMPIRAN B

PENURUNAN PERSAMAAN MAXWELL II

Tensor kuat-medan elektromagnetik diberikan oleh (A.15) di mana determinannya

$$det\,\left|F_{\alpha\beta}\right| = 0 + e^{\phi(r)}E_r \begin{bmatrix} e^{\phi(r)}E_r & e^{\Lambda(r)}r^2 sin\,\theta\,B^{\varphi} & -e^{\Lambda(r)}r^2 sin\,\theta\,B^{\theta} \\ e^{\phi(r)}E_\theta & 0 & e^{\Lambda(r)}r^2 sin\,\theta\,B^{r} \\ e^{\phi(r)}E_\varphi & -e^{\Lambda(r)}r^2 sin\,\theta\,B^{r} & 0 \end{bmatrix}$$

$$-e^{\phi(r)}E_\theta \begin{bmatrix} e^{\phi(r)}E_r & 0 & -e^{\Lambda(r)}r^2 sin\,\theta\,B^{\theta} \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\,\theta\,B^{\varphi} & e^{\Lambda(r)}r^2 sin\,\theta\,B^{r} \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\,\theta\,B^{\theta} & 0 \end{bmatrix}$$

$$+e^{\phi(r)}E_\phi \begin{bmatrix} e^{\phi(r)}E_r & 0 & e^{\Lambda(r)}r^2 sin\,\theta\,B^{\varphi} \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\,\theta\,B^{\varphi} & 0 \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\,\theta\,B^{\theta} & -e^{\Lambda(r)}r^2 sin\,\theta\,B^{r} \end{bmatrix}$$

$$det\,\left|F_{\alpha\beta}\right| = e^{\phi(r)}E_r\Big\{e^{\phi(r)}E_r\big[0 + e^{2\Lambda(r)}(r^2 sin\,\theta\,B^r)^2\big]$$
$$- e^{\Lambda(r)}r^2 sin\,\theta\,B^{\varphi}\big[0$$
$$- e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,B^r E_\varphi\big] - e^{\Lambda(r)}r^2 sin\,\theta\,B^{\theta}\big[-e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,B^r E_\theta$$
$$- 0\big]\Big\}$$
$$-e^{\phi(r)}E_\theta\Big\{e^{\phi(r)}E_r\big[0 - e^{2\Lambda(r)}(r^2 sin\,\theta\,)^2 B^r B^{\theta}\big]$$
$$- 0 - e^{\Lambda(r)}r^2 sin\,\theta\,B^{\theta}\big[e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,B^{\theta}E_\theta$$
$$+ e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,B^{\varphi}E_\varphi\big]\Big\}$$
$$+e^{\phi(r)}E_\varphi\Big\{e^{\phi(r)}E_r\big[e^{2\Lambda(r)}(r^2 sin\,\theta\,)^2 B^{\varphi}B^r - 0\big] - 0$$
$$+ e^{\Lambda(r)}r^2 sin\,\theta\,B^{\varphi}\big[e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,B^{\theta}E_\theta$$
$$+ e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,B^{\varphi}E_\varphi\big]\Big\}$$

$$det\,|F_{\alpha\beta}| = e^{\phi(r)}E_r\{e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta\,B^r)^2E_r + e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^r E_\varphi - e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2B^\theta B^r E_\theta\}$$

$$-e^{\phi(r)}E_\theta\{-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2B^r B^\theta E_r - e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2B^\theta B^\theta E_\theta - e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2B^\theta B^\varphi E_\varphi\}$$

$$+e^{\phi(r)}E_\varphi\{e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2E_r B^\varphi B^r + e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^\theta E_\theta + e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^\varphi E_\varphi\}$$

$$det\,|F_{\alpha\beta}| = \{e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta\,)^2B^r B^r E_r E_r + e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^r E_\varphi E_r + e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\theta B^r E_\theta E_r\}$$

$$+\{e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^r B^\theta E_r E_\theta + e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta\,)^2B^\theta B^\theta E_\theta E_\theta + e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\theta B^\varphi E_\theta E_\varphi\}$$

$$+\{e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^r E_\varphi E_r + e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^\theta E_\varphi E_\theta + e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^\varphi E_\varphi E_\varphi\}$$

$$det\,|F_{\alpha\beta}| = e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta\,E_r)^2 + e^{2[\phi+\Lambda](r)}\left(r^2 sin\,\theta\,B^\theta E_\theta\right)^2 + e^{2[\phi+\Lambda](r)}\left(r^2 sin\,\theta\,B^\varphi E_\varphi\right)^2 + 2e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^r E_\varphi E_r + 2e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^\varphi B^\theta E_\varphi E_\theta + 2e^{2[\phi+\Lambda](r)}(r^2 sin\,\theta)^2B^r B^\theta E_r E_\theta$$

$$det\,|F_{\alpha\beta}| = \left[e^{[\phi+\Lambda](r)}r^2 sin\,\theta\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)\right]^2 \qquad \text{(B.1)}$$

Matrik kofaktor dari tensor kuat-medan elektromagnetik $F_{\alpha\beta}$ adalah

$$C = \begin{bmatrix} C_{00} & C_{01} & C_{02} & C_{03} \\ C_{10} & C_{11} & C_{12} & C_{13} \\ C_{20} & C_{21} & C_{22} & C_{23} \\ C_{30} & C_{31} & C_{32} & C_{33} \end{bmatrix}$$

(B.2)

dengan nilai tiap-tiap komponennya

5. Baris pertama

$$C_{00} = \begin{bmatrix} 0 & e^{\Lambda(r)} r^2 sin\,\theta\; B^{\varphi} & -e^{\Lambda(r)} r^2 sin\,\theta\; B^{\theta} \\ -e^{\Lambda(r)} r^2 sin\,\theta\; B^{\varphi} & 0 & e^{\Lambda(r)} r^2 sin\,\theta\; B^{r} \\ e^{\Lambda(r)} r^2 sin\,\theta\; B^{\theta} & -e^{\Lambda(r)} r^2 sin\,\theta\; B^{r} & 0 \end{bmatrix}$$

$$C_{00} = \left[0 + e^{3\Lambda(r)}(r^2 sin\,\theta) B^{\varphi} B^{r} B^{\theta} - e^{3\Lambda(r)}(r^2 sin\,\theta) B^{\theta} B^{r} B^{\varphi}\right] - [0 + 0 + 0] = 0$$

(B.3)

$$C_{01} = -\begin{bmatrix} e^{\phi(r)} E_r & e^{\Lambda(r)} r^2 sin\,\theta\; B^{\varphi} & -e^{\Lambda(r)} r^2 sin\,\theta\; B^{\theta} \\ e^{\phi(r)} E_\theta & 0 & e^{\Lambda(r)} r^2 sin\,\theta\; B^{r} \\ e^{\phi(r)} E_\varphi & -e^{\Lambda(r)} r^2 sin\,\theta\; B^{r} & 0 \end{bmatrix}$$

$$\begin{aligned} C_{01} = -\Big\{&\left[0 + e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^{\varphi} B^{r} E_{\varphi} + e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^{\theta} B^{r} E_{\theta}\right] \\ &- \left[0 + 0 - e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^{r} B^{r} E_{r}\right]\Big\} \\ &= -e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^{r}\left(B^{r} E_{r} + B^{\theta} E_{\theta} + B^{\varphi} E_{\varphi}\right) \end{aligned}$$

(B.4)

$$C_{02} = \begin{bmatrix} e^{\phi(r)} E_r & 0 & -e^{\Lambda(r)} r^2 sin\,\theta\; B^{\theta} \\ e^{\phi(r)} E_\theta & -e^{\Lambda(r)} r^2 sin\,\theta\; B^{\varphi} & e^{\Lambda(r)} r^2 sin\,\theta\; B^{r} \\ e^{\phi(r)} E_\varphi & e^{\Lambda(r)} r^2 sin\,\theta\; B^{\theta} & 0 \end{bmatrix}$$

$$C_{02} = \left[0 + 0 - e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\theta B^\theta E_\theta\right]$$
$$- \left[e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\theta B^\varphi E_\varphi + 0\right.$$
$$\left. + e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\theta B^r E_r\right]$$
$$= -e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\theta \left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)$$

(B.5)

$$C_{03} = -\begin{bmatrix} e^{\phi(r)}E_r & 0 & e^{\Lambda(r)}r^2 sin\ \theta\ B^\varphi \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\ \theta\ B^\varphi & 0 \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\ \theta\ B^\theta & -e^{\Lambda(r)}r^2 sin\ \theta\ B^r \end{bmatrix}$$

$$C_{03} = -\left\{\left[e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi B^r E_r + 0 + e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi B^\theta E_\theta\right]\right.$$
$$\left. - \left[-e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi B^\varphi E_\varphi + 0 + 0\right]\right\}$$
$$= -e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi \left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)$$

(B.6)

6. Baris kedua

$$C_{01} = -\begin{bmatrix} -e^{\phi(r)}E_r & -e^{\phi(r)}E_\theta & -e^{\phi(r)}E_\varphi \\ -e^{\Lambda(r)}r^2 sin\ \theta\ B^\varphi & 0 & e^{\Lambda(r)}r^2 sin\ \theta\ B^r \\ e^{\Lambda(r)}r^2 sin\ \theta B^\theta & -e^{\Lambda(r)}r^2 sin\ \theta\ B^r & 0 \end{bmatrix}$$

$$C_{01} = -\left\{\left[0 - e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\theta B^r E_\theta - e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^r B^\varphi E_\varphi\right]\right.$$
$$\left. - \left[0 + 0 + e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^r B^r E_r\right]\right\}$$
$$= -e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^r \left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)$$

(B.7)

$$C_{11} = \begin{bmatrix} 0 & -e^{\phi(r)}E_\theta & -e^{\phi(r)}E_\varphi \\ e^{\phi(r)}E_\theta & 0 & e^{\Lambda(r)}r^2 sin\ \theta\ B^r \\ -e^{\phi(r)}E_\varphi & -e^{\Lambda(r)}r^2 sin\ \theta\ B^r & 0 \end{bmatrix}$$

$$C_{11} = \left[0 - e^{[2\phi+\Lambda](r)}(r^2 sin\,\theta\,)B^r E_\theta E_\varphi + e^{[2\phi+\Lambda](r)}(r^2 sin\,\theta\,)B^r E_\theta E_\varphi\right]$$
$$- [0 + 0 + 0] = 0$$

(B.8)

$$C_{12} = -\begin{bmatrix} 0 & -e^{\phi(r)}E_r & -e^{\phi(r)}E_\varphi \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\,\theta\,B^\varphi & e^{\Lambda(r)}r^2 sin\,\theta B^r \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\,\theta B^\theta & 0 \end{bmatrix}$$

$$C_{12} = -\left\{\left[0 - e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\,B^r E_r E_\varphi - e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\,B^\theta E_\theta E_\varphi\right]\right.$$
$$\left. - \left[e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\,B^\varphi E_\varphi E_\varphi + 0 + 0\right]\right\}$$
$$= e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\,E_\varphi\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)$$

(B.9)

$$C_{13} = \begin{bmatrix} 0 & -e^{\phi(r)}E_r & -e^{\phi(r)}E_\theta \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\,\theta\,B^\varphi & 0 \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\,\theta B^\theta & -e^{\Lambda(r)}r^2 sin\,\theta\,B^r \end{bmatrix}$$

$$C_{13} = \left[0 + 0 - e^{[2\phi+\Lambda](r)}(r^2 sin\,\theta\,)B^\theta E_\theta E_\theta\right]$$
$$- \left[e^{[2\phi+\Lambda](r)}(r^2 sin\,\theta\,)B^\varphi E_\varphi E_\theta + e^{[2\phi+\Lambda](r)}(r^2 sin\,\theta\,)B^r E_r E_\theta + 0\right]$$
$$= -e^{[2\phi+\Lambda](r)}r^2 sin\,\theta - \;\theta\,E_\theta\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)$$

(B.10)

7. Baris ketiga

$$C_{20} = \begin{bmatrix} -e^{\phi(r)}E_r & -e^{\phi(r)}E_\theta & -e^{\phi(r)}E_\varphi \\ 0 & e^{\Lambda(r)}r^2 sin\,\theta\,B^\varphi & -e^{\Lambda(r)}r^2 sin\,\theta B^\theta \\ e^{\Lambda(r)}r^2 sin\,\theta B^\theta & -e^{\Lambda(r)}r^2 sin\,\theta\,B^r & 0 \end{bmatrix}$$

$$
\begin{aligned}
C_{20} = &\left[0 + e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\theta B^\theta E_\theta + 0\right] \\
&- \left[-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\theta B^\varphi E_\varphi + 0 \right.\\
&\left.- e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\theta B^r E_r\right] \\
&= e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\theta\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)
\end{aligned}
\tag{B.11}
$$

$$
C_{21} = -\begin{bmatrix} 0 & -e^{\phi(r)}E_\theta & -e^{\phi(r)}E_\varphi \\ e^{\phi(r)}E_r & e^{\Lambda(r)}r^2 sin\,\theta\, B^\varphi & -e^{\Lambda(r)}r^2 sin\,\theta B^\theta \\ e^{\phi(r)}E_\varphi & -e^{\Lambda(r)}r^2 sin\,\theta B^r & 0 \end{bmatrix}
$$

$$
\begin{aligned}
C_{21} = &-\left\{\left[0 + e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\theta E_\theta E_\varphi + e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^r E_r E_\varphi\right]\right. \\
&\left.- \left[-e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\varphi E_\varphi E_\varphi + 0 + 0\right]\right\} \\
&= -e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, E_\varphi\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)
\end{aligned}
\tag{B.12}
$$

$$
C_{22} = \begin{bmatrix} 0 & -e^{\phi(r)}E_r & -e^{\phi(r)}E_\varphi \\ e^{\phi(r)}E_r & 0 & -e^{\Lambda(r)}r^2 sin\,\theta B^\theta \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\,\theta B^\theta & 0 \end{bmatrix}
$$

$$
\begin{aligned}
C_{22} = &\left[0 + e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\theta E_r E_\varphi - e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\theta E_r E_\varphi\right] - [0 + 0 + 0] \\
&= 0
\end{aligned}
\tag{B.13}
$$

$$
C_{23} = -\begin{bmatrix} 0 & -e^{\phi(r)}E_r & -e^{\phi(r)}E_\theta \\ e^{\phi(r)}E_r & 0 & e^{\Lambda(r)}r^2 sin\,\theta B^\varphi \\ e^{\phi(r)}E_\varphi & e^{\Lambda(r)}r^2 sin\,\theta B^\theta & -e^{\Lambda(r)}r^2 sin\,\theta B^r \end{bmatrix}
$$

$$
\begin{aligned}
C_{23} = &-\left\{\left[0 - e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\varphi E_\varphi E_r - e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\theta E_\theta E_r\right]\right. \\
&\left.- \left[0 + e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^r E_r E_r + 0\right]\right\} \\
&= e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, E_r\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)
\end{aligned}
\tag{B.14}
$$

8. Baris keempat

$$C_{30} = -\begin{bmatrix} -e^{\phi(r)}E_r & -e^{\phi(r)}E_\theta & -e^{\phi(r)}E_\varphi \\ 0 & e^{\Lambda(r)}r^2 sin\ \theta B^\varphi & -e^{\Lambda(r)}r^2 sin\ \theta B^\theta \\ -e^{\Lambda(r)}r^2 sin\ \theta B^\varphi & 0 & e^{\Lambda(r)}r^2 sin\ \theta B^r \end{bmatrix}$$

$$\begin{aligned} C_{30} = -\{&\left[-e^{[\phi+2\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi B^r E_r - e^{[2\phi+\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi B^\theta E_\theta + 0\right] \\ &- \left[e^{[2\phi+\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi B^\varphi E_\varphi + 0 + 0\right]\} \\ &= -e^{[2\phi+\Lambda](r)}(r^2 sin\ \theta)^2 B^\varphi \left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right) \end{aligned}$$

(B.15)

$$C_{31} = \begin{bmatrix} 0 & -e^{\phi(r)}E_\theta & -e^{\phi(r)}E_\varphi \\ e^{\phi(r)}E_r & e^{\Lambda(r)}r^2 sin\ \theta\ B^\varphi & -e^{\Lambda(r)}r^2 sin\ \theta B^\theta \\ e^{\phi(r)}E_\theta & 0 & e^{\Lambda(r)}r^2 sin\ \theta B^r \end{bmatrix}$$

$$\begin{aligned} C_{31} = &\left[0 + e^{[2\phi+\Lambda](r)}r^2 sin\ \theta\ B^\theta E_\theta E_\theta + 0\right] \\ &- \left[-e^{e^{[2\phi+\Lambda](r)}}r^2 sin\ \theta\ B^\varphi E_\varphi E_\theta - e^{[2\phi+\Lambda](r)}r^2 sin\ \theta\ B^r E_r E_\theta + 0\right] \\ &= e^{[2\phi+\Lambda](r)}r^2 sin\ \theta\ E_\theta\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right) \end{aligned}$$

(B.16)

$$C_{32} = -\begin{bmatrix} 0 & -e^{\phi(r)}E_r & -e^{\phi(r)}E_\varphi \\ e^{\phi(r)}E_r & 0 & -e^{\Lambda(r)}r^2 sin\ \theta B^\theta \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\ \theta B^\varphi & e^{\Lambda(r)}r^2 sin\ \theta B^r \end{bmatrix}$$

$$\begin{aligned} C_{32} = -\{&\left[0 + e^{[2\phi+\Lambda](r)}r^2 sin\ \theta\ B^\theta E_\theta E_r + e^{[2\phi+\Lambda](r)}r^2 sin\ \theta\ B^\varphi E_\varphi E_r\right] \\ &- \left[0 - e^{[2\phi+\Lambda](r)}r^2 sin\ \theta\ B^r E_r E_r + 0\right]\} \\ &= -e^{[2\phi+\Lambda](r)}r^2 sin\ \theta\ E_r\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right) \end{aligned}$$

(B.17)

$$C_{33}=\begin{bmatrix}0 & -e^{\phi(r)}E_r & -e^{\phi(r)}E_\theta \\ e^{\phi(r)}E_r & 0 & e^{\Lambda(r)}r^2 sin\,\theta B^\varphi \\ e^{\phi(r)}E_\theta & -e^{\Lambda(r)}r^2 sin\,\theta B^\varphi & 0\end{bmatrix}$$

$$C_{33}=-\left\{\left[0-e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\varphi E_\theta E_r+e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, B^\varphi E_\theta E_r\right]-[0+0+0]\right\}=0$$

(B.18)

Maka transpos dari matrik kovaktornya diberikan oleh

$$C^T=\begin{bmatrix}C_{00} & C_{10} & C_{20} & C_{30} \\ C_{01} & C_{11} & C_{21} & C_{31} \\ C_{02} & C_{12} & C_{22} & C_{32} \\ C_{03} & C_{13} & C_{23} & C_{33}\end{bmatrix}=\left(B^r E_r+B^\theta E_\theta+B^\varphi E_\varphi\right)\begin{bmatrix}C_{00}' & C_{10}' & C_{20}' & C_{30}' \\ C_{01}' & C_{11}' & C_{21}' & C_{31}' \\ C_{02}' & C_{12}' & C_{22}' & C_{32}' \\ C_{03}' & C_{13}' & C_{23}' & C_{33}'\end{bmatrix}$$

(B.19)

dengan

$$C_{00}'=C_{11}'=C_{22}'=C_{33}'=0$$
$$C_{01}'=-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^r=-C_{10}'$$
$$C_{02}'=-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\theta=-C_{20}'$$
$$C_{03}'=-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\varphi=-C_{30}'$$
$$C_{12}'=e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, E_\varphi=-C_{21}'$$
$$C_{13}'=-e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, E_\theta=-C_{31}'$$
$$C_{32}'=-e^{[2\phi+\Lambda](r)}r^2 sin\,\theta\, E_r=-C_{23}'$$

(B.20)

sedemikian rupa sehingga tensor kuat-medan elektromagnetik kontravarian adalah

$$F^{\alpha\beta} = \frac{C^T}{det\left|F_{\alpha\beta}\right|}$$

$$= \frac{\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)}{\left[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)\right]^2}\begin{bmatrix} C_{00}{}' & C_{10}{}' & C_{20}{}' & C_{30}{}' \\ C_{01}{}' & C_{11}{}' & C_{21}{}' & C_{31}{}' \\ C_{02}{}' & C_{12}{}' & C_{22}{}' & C_{32}{}' \\ C_{03}{}' & C_{13}{}' & C_{23}{}' & C_{33}{}' \end{bmatrix}$$

(B.20)

Dengan menuliskan $\beta = \left[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,\right]^2$ dan memasukkan niai ini ke dalam matriks (B.21), maka didapat

$$F^{\alpha\beta} = \frac{1}{\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right)}\begin{bmatrix} \frac{C_{00}{}'}{\beta} & \frac{C_{10}{}'}{\beta} & \frac{C_{20}{}'}{\beta} & \frac{C_{30}{}'}{\beta} \\ \frac{C_{01}{}'}{\beta} & \frac{C_{11}{}'}{\beta} & \frac{C_{21}{}'}{\beta} & \frac{C_{31}{}'}{\beta} \\ \frac{C_{02}{}'}{\beta} & \frac{C_{12}{}'}{\beta} & \frac{C_{22}{}'}{\beta} & \frac{C_{32}{}'}{\beta} \\ \frac{C_{03}{}'}{\beta} & \frac{C_{13}{}'}{\beta} & \frac{C_{23}{}'}{\beta} & \frac{C_{33}{}'}{\beta} \end{bmatrix}$$

(B.21)

dengan

$$F'^{00} = \frac{C_{00}{}'}{\beta} = 0$$

$$F'^{11} = \frac{C_{11}{}'}{\beta} = 0$$

$$F'^{22} = \frac{C_{22}{}'}{\beta} = 0$$

$$F'^{33} = \frac{C_{33}{}'}{\beta} = 0$$

$$F'^{01} = \frac{C_{01}{}'}{\beta} = \frac{-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^r}{[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,]^2} = -e^{-\phi(r)} B^r = -\frac{C_{10}{}'}{\beta} = -F'^{10}$$

$$F'^{02} = \frac{C_{02}{}'}{\beta} = \frac{-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\theta}{[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,]^2} = -e^{-\phi(r)} B^\theta = -\frac{C_{20}{}'}{\beta} = -F'^{20}$$

$$F'^{03} = \frac{C_{03}{}'}{\beta} = \frac{-e^{[\phi+2\Lambda](r)}(r^2 sin\,\theta)^2 B^\varphi}{[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,]^2} = -e^{-\phi(r)} B^\varphi = -\frac{C_{30}{}'}{\beta} = -F'^{30}$$

$$F'^{12} = \frac{C_{12}{}'}{\beta} = \frac{e^{[2\phi+\Lambda](r)} r^2 sin\,\theta\, E_\varphi}{[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,]^2} = e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} E_\varphi = -\frac{C_{21}{}'}{\beta} = -F'^{21}$$

$$F'^{13} = \frac{C_{13}{}'}{\beta} = \frac{-e^{[2\phi+\Lambda](r)} r^2 sin\,\theta\, E_\theta}{[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,]^2} = -e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} E_\theta = -\frac{C_{31}{}'}{\beta} = -F'^{31}$$

$$F'^{32} = \frac{C_{32}{}'}{\beta} = \frac{-e^{[2\phi+\Lambda](r)} r^2 sin\,\theta\, E_r}{[e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,]^2} = -e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} E_r = -\frac{C_{23}{}'}{\beta} = -F'^{23}$$

dan

$$\left(B^r E_r + B^\theta E_\theta + B^\varphi E_\varphi\right) = \left(B^i E_i\right)$$

maka persamaan (B.21) menjadi

$$F^{\alpha\beta} = \frac{1}{(B^i E_i)} \begin{bmatrix} F'^{00} & F'^{10} & F'^{20} & F'^{30} \\ F'^{01} & F'^{11} & F'^{21} & F'^{31} \\ F'^{02} & F'^{12} & F'^{22} & F'^{32} \\ F'^{03} & F'^{13} & F'^{23} & F'^{33} \end{bmatrix}. \qquad (B.22)$$

Dengan mensubstitusikan persamaan (B.22) ke persamaan (2.135), maka didapatkan medan listrik, yaitu

$$E^\mu = F^{\mu v} u_v. \qquad (B.23)$$

Mengingat kecepatan-4 $u_0 = e^{\phi(r)}$, $u_1 = 0$, $u_2 = 0$ dan $u_3 = 0$, maka komponen medan listriknya adalah

4. Komponen radial

$$E^r = E^1 = F^{rv}u_v = F^{10}u_0 + F^{11}u_1 + F^{12}u_2 + F^{13}u_3 = F^{10}u_0$$

$$E^r = \frac{e^{-\phi(r)}B^r}{(B^iE_i)}e^{\phi(r)}$$

$$E^r = \frac{B^r}{(B^iE_i)} \tag{B.24}$$

5. Komponen polar

$$E^\theta = E^2 = F^{\theta v}u_v = F^{20}u_0 + F^{21}u_1 + F^{22}u_2 + F^{23}u_3 = F^{20}u_0$$

$$E^\theta = \frac{e^{-\phi(r)}B^\theta}{(B^iE_i)}e^{\phi(r)}$$

$$E^\theta = \frac{B^\theta}{(B^iE_i)} \tag{B.25}$$

6. Komponen toroidal

$$E^\varphi = E^3 = F^{\varphi v}u_v = F^{30}u_0 + F^{31}u_1 + F^{32}u_2 + F^{33}u_3 = F^{30}u_0$$

$$E^{\varphi} = \frac{e^{-\phi(r)}B^{\varphi}}{(B^iE_i)}e^{\phi(r)}$$

$$E^{\varphi} = \frac{B^{\varphi}}{(B^iE_i)}$$

(B.26)

Selanjutnya diketahui dari persamaan (2.136)

$$B_{\alpha} = F^{\cdot}_{\alpha\beta}u^{\beta} = \frac{1}{2}\eta_{\alpha\beta\gamma\delta}u^{\beta}F^{\gamma\delta} = \frac{1}{2}\sqrt{-g}\,\epsilon_{\alpha\beta\gamma\delta}u^{\beta}F^{\gamma\delta}$$

$$B_{\alpha} = \frac{1}{2}e^{[\phi+\Lambda](r)}r^2 sin\,\theta\;\;\epsilon_{\alpha\beta\gamma\delta}u^{\beta}F^{\gamma\delta}$$

(B.27)

Maka komponen medan magnetnya

4. Komponen radial

$$B_r = B_1 = \frac{1}{2}e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,(\epsilon_{1023}u^0F^{23} + \epsilon_{1032}u^0F^{32} + \epsilon_{1203}u^2F^{03} + \epsilon_{1230}u^2F^{30} + \epsilon_{1302}u^3F^{02} + \epsilon_{1320}u^3F^{20})$$

$$B_r = \frac{1}{2}e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,(\epsilon_{1023}u^0F^{23} + \epsilon_{1032}u^0F^{32})$$

Dengan $\epsilon_{1023} = -\epsilon_{0123} = -1$ dan $\epsilon_{1032} = -\epsilon_{1023} = -(-\epsilon_{0123}) = +1$, maka

$$B_r = \frac{1}{2}e^{[\phi+\Lambda](r)}r^2 sin\,\theta\;\frac{e^{-\phi(r)}}{(B^iE_i)}\left(-e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}E_r - e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}E_r\right)$$

$$B_r = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \frac{e^{-\phi(r)}}{(B^i E_i)} \left(-2e^{-\Lambda(r)} (r^2 sin\,\theta\,)^{-1} E_r\right)$$

$$B_r = -\frac{E_r}{(B^i E_i)}$$

(B.28)

5. Komponen polar

$$B_\theta = B_2 = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ (\epsilon_{2013} u^0 F^{13} + \epsilon_{2031} u^0 F^{31} + \epsilon_{2103} u^1 F^{03} + \epsilon_{2130} u^1 F^{30} + \epsilon_{2301} u^3 F^{01} + \epsilon_{2310} u^3 F^{10})$$

$$B_r = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ (\epsilon_{2013} u^0 F^{13} + \epsilon_{2031} u^0 F^{31})$$

Dengan $\epsilon_{2013} = -\epsilon_{0213} = -(-\epsilon_{0123}) = +1$ dan $\epsilon_{2031} = -\epsilon_{0231} = -(-\epsilon_{0132}) = -\epsilon_{0123} = -1$, maka

$$B_\theta = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \frac{e^{-\phi(r)}}{(B^i E_i)} \left(-e^{-\Lambda(r)} (r^2 sin\,\theta\,)^{-1} E_\theta - e^{-\Lambda(r)} (r^2 sin\,\theta\,)^{-1} E_\theta\right)$$

$$B_\theta = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \frac{e^{-\phi(r)}}{(B^i E_i)} \left(-2e^{-\Lambda(r)} (r^2 sin\,\theta\,)^{-1} E_\theta\right)$$

$$B_\theta = -\frac{E_\theta}{(B^i E_i)}$$

(B.29)

6. Komponen toroidal

$$B_\varphi = B_3 = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ (\epsilon_{3012} u^0 F^{12} + \epsilon_{3021} u^0 F^{21} + \epsilon_{3102} u^1 F^{02} + \epsilon_{3120} u^1 F^{20} + \epsilon_{3201} u^2 F^{01} + \epsilon_{3210} u^2 F^{10})$$

$$B_\varphi = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ (\epsilon_{3012} u^0 F^{12} + \epsilon_{3021} u^0 F^{21})$$

Dengan $\epsilon_{3012} = -\epsilon_{0312} = -(-\epsilon_{0132}) = -\epsilon_{0123} = -1$ dan $\epsilon_{3021} = -\epsilon_{0321} = -(-\epsilon_{0123}) = +1$, maka

$$B_\varphi = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \frac{e^{-\phi(r)}}{(B^i E_i)} \left(-e^{-\Lambda(r)} (r^2 sin\,\theta\,)^{-1} E_\varphi - e^{-\Lambda(r)} (r^2 sin\,\theta\,)^{-1} E_\varphi\right)$$

$$B_\varphi = \frac{1}{2} e^{[\phi+\Lambda](r)} r^2 sin\,\theta -\ \theta\ \frac{e^{-\phi(r)}}{(B^i E_i)} \left(-2 e^{-\Lambda(r)} (r^2 sin\,\theta\,)^{-1} E_\varphi\right)$$

$$B_\varphi = -\frac{E_\varphi}{(B^i E_i)}$$

(B.30)

Oleh karena itu, tensor kuat-medan elektromagnetik kontravarian adalah

$$F^{\alpha\beta} = \begin{bmatrix} F^{00} & F^{10} & F^{20} & F^{30} \\ F^{01} & F^{11} & F^{21} & F^{31} \\ F^{02} & F^{12} & F^{22} & F^{32} \\ F^{03} & F^{13} & F^{23} & F^{33} \end{bmatrix}$$

(B.31)

dengan

$$F^{00} = F^{11} = F^{22} = F^{33} = 0$$

$$F^{01} = \frac{F'^{01}}{(B^i E_i)} = \frac{e^{-\phi(r)} B_r}{(B^i E_i)} = e^{-\phi(r)} E^r = -F^{10}$$

$$F^{02} = \frac{F'^{02}}{(B^i E_i)} = \frac{e^{-\phi(r)} B_\theta}{(B^i E_i)} = e^{-\phi(r)} E^\theta = -F^{20}$$

$$F^{03} = \frac{F'^{03}}{(B^i E_i)} = \frac{e^{-\phi(r)} B_\varphi}{(B^i E_i)} = e^{-\phi(r)} E^\varphi = -F^{30}$$

$$F^{12} = \frac{F'^{12}}{(B^i E_i)} = \frac{e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} E_\varphi}{(B^i E_i)} = -e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} B_\varphi = -F^{21}$$

$$F^{13} = \frac{F'^{13}}{(B^i E_i)} = \frac{-e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} E_\theta}{(B^i E_i)} = e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} B_\theta = -F^{31}$$

$$F^{32} = \frac{F'^{32}}{(B^i E_i)} = \frac{-e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} E_r}{(B^i E_i)} = e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1} B_r = -F^{23}$$

(B.32)

Dari persamaan (2.133), komponen rapat arus-4 adalah

$$J^\alpha = \rho_e u^\alpha + j^\alpha = \rho_e u^\alpha + \sigma F_{\alpha\beta} u^\beta$$

(B.33)

5. Komponen waktu

$$J^t = \rho_e u^0 + \sigma(F_{00}u^0 + F_{01}u^1 + F_{02}u^2 + F_{03}u^3) = \rho_e u^0 + \sigma F_{00} u^0 = \rho_e u^0$$
$$= \rho_e e^{-\phi(r)}$$

(B.34)

6. Komponen radial

$$J^r = \rho_e u^1 + \sigma(F_{10}u^0 + F_{11}u^1 + F_{12}u^2 + F_{13}u^3) = \sigma F_{10} u^0 = \sigma e^{\phi(r)} E_r e^{-\phi(r)}$$
$$= \sigma E_r$$

(B.35)

7. Komponen polar

$$J^\theta = \rho_e u^2 + \sigma(F_{20}u^0 + F_{21}u^1 + F_{22}u^2 + F_{23}u^3) = \sigma F_{20} u^0 = \sigma e^{\phi(r)} E_\theta e^{-\phi(r)}$$
$$= \sigma E_\theta$$

(B.36)

8. Komponen toroidal

$$J^{\varphi} = \rho_e u^3 + \sigma(F_{30}u^0 + F_{31}u^1 + F_{32}u^2 + F_{33}u^3) = \sigma F_{30}u^0 = \sigma e^{\phi(r)} E_{\varphi} e^{-\phi(r)}$$
$$= \sigma E_{\varphi}$$

(B.37)

Dari semua hasil di atas, maka Persamaan Maxwell II pada (2.132) dapat diturunkan

$$F^{\alpha\beta}_{;\beta} = \frac{1}{\sqrt{-g}}\left(\sqrt{-g}\, F^{\alpha\beta}\right)_{,\beta} = 4\pi J^{\alpha}$$

atau

$$F^{\alpha\beta}_{;\beta} = \left(\sqrt{-g}\, F^{\alpha\beta}\right)_{,\beta} = \sqrt{-g}\, 4\pi J^{\alpha}$$

(B.38)

5. Untuk $\alpha = 0$

$$F^{00}_{;0} + F^{01}_{;1} + F^{02}_{;2} + F^{03}_{;3} = \sqrt{-g}\, 4\pi J^0$$
$$\left(\sqrt{-g}\, F^{00}\right)_{,0} + \left(\sqrt{-g}\, F^{01}\right)_{,1} + \left(\sqrt{-g}\, F^{02}\right)_{,2} + \left(\sqrt{-g}\, F^{03}\right)_{,3} = \sqrt{-g}\, 4\pi J^0$$
$$\left[\left(e^{[\phi+\Lambda](r)} r^2 sin\,\theta\right)0\right]_{,t} + \left[\left(e^{[\phi+\Lambda](r)} r^2 sin\,\theta\right) e^{-\phi(r)} E^r\right]_{,r}$$
$$+ \left[\left(e^{[\phi+\Lambda](r)} r^2 sin\,\theta\right) e^{-\phi(r)} E^{\theta}\right]_{,\theta}$$
$$+ \left[\left(e^{[\phi+\Lambda](r)} r^2 sin\,\theta\right) e^{-\phi(r)} E^{\varphi}\right]_{,\varphi} = 4\pi\, e^{[\phi+\Lambda](r)} r^2 sin\,\theta\, J^t$$
$$\left[e^{\Lambda(r)} r^2 sin\,\theta E^r\right]_{,r} + \left[e^{\Lambda(r)} r^2 sin\,\theta E^{\theta}\right]_{,\theta} + \left[e^{\Lambda(r)} r^2 sin\,\theta E^{\varphi}\right]_{,\varphi}$$
$$= 4\pi\, e^{[\phi+\Lambda](r)} r^2 sin\,\theta\, J^t$$
$$\left[e^{\Lambda(r)} r^2 sin\,\theta E^i\right]_{,i} = 4\pi\, e^{[\phi+\Lambda](r)} r^2 sin\,\theta\, J^t$$

(B.39)

6. Untuk $\alpha = 1$

$$F^{10}_{;0} + F^{11}_{;1} + F^{12}_{;2} + F^{13}_{;3} = \sqrt{-g}\ 4\pi J^1$$

$$\left(\sqrt{-g}\ F^{10}\right)_{,0} + \left(\sqrt{-g}\ F^{11}\right)_{,1} + \left(\sqrt{-g}\ F^{12}\right)_{,2} + \left(\sqrt{-g}\ F^{13}\right)_{,3} = \sqrt{-g}\ 4\pi J^1$$

$$\left[-\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) e^{-\phi(r)} E^r\right]_{,t} + \left[\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) 0\right]_{,r}$$
$$+ \left[-\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) e^{-\Lambda(r)} (r^2 sin\ \theta\ )^{-1} B_\varphi\right]_{,\theta}$$
$$+ \left[\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) e^{-\Lambda(r)} (r^2 sin\ \theta\ )^{-1} B_\theta\right]_{,\varphi}$$
$$= 4\pi e^{[\phi+\Lambda](r)} r^2 sin\ \theta\ J^r$$

$$\left[-e^{\Lambda(r)} r^2 sin\ \theta E^r\right]_{,t} + \left[-e^{\phi(r)} B_\varphi\right]_{,\theta} + \left[e^{\phi(r)} B_\theta\right]_{,\varphi} = 4\pi e^{[\phi+\Lambda](r)} r^2 sin\ \theta\ J^r$$

$$\left[-e^{\phi(r)} B_\phi\right]_{,\theta} + \left[e^{\phi(r)} B_\theta\right]_{,\varphi} = \left[e^{\Lambda(r)} r^2 sin\ \theta E^r\right]_{,t} + 4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\ \theta\ J^r$$

$$e^{\phi(r)}\left(B_{\theta,\varphi} - B_{\varphi,\theta}\right) = e^{\Lambda(r)} r^2 sin\ \theta\ \frac{\partial E^r}{\partial t} + 4\pi e^{[\phi+\Lambda](r)} r^2 sin\ \theta\ J^r \tag{B.40}$$

7. Untuk $\alpha = 2$

$$F^{20}_{;0} + F^{21}_{;1} + F^{22}_{;2} + F^{23}_{;3} = \sqrt{-g}\ 4\pi J^2$$

$$\left(\sqrt{-g}\ F^{20}\right)_{,0} + \left(\sqrt{-g}\ F^{21}\right)_{,1} + \left(\sqrt{-g}\ F^{22}\right)_{,2} + \left(\sqrt{-g}\ F^{23}\right)_{,3} = \sqrt{-g}\ 4\pi J^2$$

$$\left[-\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) e^{-\phi(r)} E^\theta\right]_{,t}$$
$$+ \left[\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) e^{-\Lambda(r)} (r^2 sin\ \theta\ )^{-1} B_\varphi\right]_{,r}$$
$$+ \left[\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) 0\right]_{,\theta}$$
$$+ \left[-\left(e^{[\phi+\Lambda](r)} r^2 sin\ \theta\right) e^{-\Lambda(r)} (r^2 sin\ \theta\ )^{-1} B_r\right]_{,\varphi}$$
$$= 4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\ \theta\ J^\theta$$

$$\left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\phi(r)}E^{\theta}\right]_{,t}$$

$$+\left[\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{\varphi}\right]_{,r}$$

$$+\left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{r}\right]_{,\varphi}$$

$$= 4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^{\theta}$$

$$\left[\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{\varphi}\right]_{,r}$$

$$+\left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{r}\right]_{,\varphi}$$

$$= \left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\phi(r)}E^{\theta}\right]_{,t} + 4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^{\theta}$$

$$\left[e^{\phi(r)}B_{\varphi}\right]_{,r} + \left[-e^{\phi(r)}B_{r}\right]_{,\varphi}$$

$$= \left[\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\phi(r)}E^{\theta}\right]_{,t} + 4\pi e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^{\theta}$$

$$e^{\phi(r)}\left(B_{\varphi,r} - B_{r,\varphi}\right) = e^{\Lambda(r)}r^2 sin\,\theta\, \frac{\partial E^{\theta}}{\partial t} + 4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^{\theta} \tag{B.41}$$

8. Untuk $\alpha = 3$

$$F^{30}_{;0} + F^{31}_{;1} + F^{32}_{;2} + F^{33}_{;3} = \sqrt{-g}\; 4\pi J^3$$

$$\left(\sqrt{-g}\; F^{30}\right)_{,0} + \left(\sqrt{-g}\; F^{31}\right)_{,1} + \left(\sqrt{-g}\; F^{32}\right)_{,2} + \left(\sqrt{-g}\; F^{33}\right)_{,3} = \sqrt{-g}\; 4\pi J^3$$

$$\left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\phi(r)}E^{\varphi}\right]_{,t}$$

$$+\left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{\theta}\right]_{,r}$$

$$+\left[\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{r}\right]_{,\theta}$$

$$+\left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)0\right]_{,\varphi} = 4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^{\varphi}$$

$$\left[-\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{\theta}\right]_{,r}$$

$$+\left[\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\Lambda(r)}(r^2 sin\,\theta\,)^{-1}B_{r}\right]_{,\theta}$$

$$= \left[\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\phi(r)}E^{\varphi}\right]_{,t} + 4\pi e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^{\varphi}$$

$$\left[-e^{\phi(r)}B_\theta\right]_{,r} + \left[e^{\phi(r)}B_r\right]_{,\theta}$$

$$= \left[\left(e^{[\phi+\Lambda](r)}r^2 sin\,\theta\right)e^{-\phi(r)}E^\theta\right]_{,t} + 4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^\varphi$$

$$e^{\phi(r)}\left(B_{r,\theta} - B_{\theta,r}\right) = e^{\Lambda(r)}r^2 sin\,\theta\, \frac{\partial E^\varphi}{\partial t} + 4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\, J^\varphi \tag{B.42}$$

LAMPIRAN C
DINAMIKA MEDAN ELEKTROMAGNETIK RELATIVISTIK BINTANG NEUTRON STASIONER

A. Medan Listrik

4. Komponen radial

Untuk medan listrik komponen radial, persamaan (B.35) disubstitusikan ke persamaan (B.40), yaitu

$$e^{\phi(r)}\left(B_{\theta,\varphi} - B_{\varphi,\theta}\right) = e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^r}{\partial t} + 4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma E_r$$

$$4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma E_r = e^{\phi(r)}\left(B_{\theta,\varphi} - B_{\varphi,\theta}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^r}{\partial t}$$

$$E_r = \frac{1}{4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma}\left[e^{\phi(r)}\left(B_{\theta,\varphi} - B_{\varphi,\theta}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^r}{\partial t}\right]$$

(C.1)

5. Komponen polar

Untuk medan listrik komponen polar, persamaan (B.36) disubstitusikan ke persamaan (B.41), yaitu

$$e^{\phi(r)}\left(B_{\varphi,r} - B_{r,\varphi}\right) = e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^\theta}{\partial t} + 4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma E_\theta$$

$$4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma E_\theta = e^{\phi(r)}\left(B_{\varphi,r} - B_{r,\varphi}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^\theta}{\partial t}$$

$$E_\theta = \frac{1}{4\pi e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma}\left[e^{\phi(r)}\left(B_{\varphi,r} - B_{r,\varphi}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^\theta}{\partial t}\right]$$

(C.2)

6. Komponen toroidal

Untuk medan listrik komponen polar, persamaan (B.37) disubstitusikan ke persamaan (B.42), yaitu

$$e^{\phi(r)}\left(B_{r,\theta} - B_{\theta,r}\right) = e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^{\varphi}}{\partial t} + 4\pi e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma E_{\varphi}$$

$$4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma E_{\varphi} = e^{\phi(r)}\left(B_{r,\theta} - B_{\theta,r}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^{\varphi}}{\partial t}$$

$$E_{\phi} = \frac{1}{4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma}\left[e^{\phi(r)}\left(B_{r,\theta} - B_{\theta,r}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^{\varphi}}{\partial t}\right]$$

(C.3)

B. Dinamika Medan Magnet

4. Komponen radial

Untuk dinamika medan magnet komponen radial, persamaan (C.2) dan (C.3) disubstitusikan ke persamaan (A.44), yaitu

$$e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial B^r}{\partial t} = e^{\phi(r)}\left(E_{\varphi}\right)_{,\theta} - e^{\phi(r)}\left(E_{\theta}\right)_{,\varphi}$$

$$e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial B^r}{\partial t}$$
$$= e^{\phi(r)}\left(\frac{1}{4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma}\left[e^{\phi(r)}\left(B_{r,\theta} - B_{\theta,r}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^{\varphi}}{\partial t}\right]\right)_{,\theta}$$
$$- e^{\phi(r)}\left(\frac{1}{4\pi\ e^{[\phi+\Lambda](r)} r^2 sin\,\theta\ \sigma}\left[e^{\phi(r)}\left(B_{\varphi,r} - B_{r,\varphi}\right) - e^{\Lambda(r)} r^2 sin\,\theta\ \frac{\partial E^{\theta}}{\partial t}\right]\right)_{,\varphi}$$

sehingga

$$\frac{\partial B^r}{\partial t} = \frac{1}{e^{\Lambda(r)} r^2 sin\,\theta} \Bigg\{ e^{\phi(r)} \Bigg(\frac{1}{4\pi\; e^{[\phi+\Lambda](r)} r^2 sin\,\theta\; \sigma} \Big[e^{\phi(r)} \big(B_{r,\theta} - B_{\theta,r} \big)$$

$$- e^{\Lambda(r)} r^2 sin\,\theta\; \frac{\partial E^{\varphi}}{\partial t} \Big] \Bigg)_{,\theta}$$

$$- e^{\phi(r)} \Bigg(\frac{1}{4\pi\; e^{[\phi+\Lambda](r)} r^2 sin\,\theta\; \sigma} \Big[e^{\phi(r)} \big(B_{\varphi,r} - B_{r,\varphi} \big)$$

$$- e^{\Lambda(r)} r^2 sin\,\theta\; \frac{\partial E^{\theta}}{\partial t} \Big] \Bigg)_{,\varphi} \Bigg\}$$

(C.4)

5. Komponen polar

Untuk dinamika medan magnet komponen polar, persamaan (C.1) dan (C.3) disubstitusikan ke persamaan (A.43), yaitu

$$e^{\Lambda(r)} r^2 sin\,\theta\; \frac{\partial B^{\theta}}{\partial t} = \big(e^{\phi(r)} E_{\varphi} \big)_{,r} - e^{\phi(r)} (E_r)_{,\varphi}$$

$$e^{\Lambda(r)} r^2 sin\,\theta\; \frac{\partial B^{\theta}}{\partial t}$$

$$= \Bigg(\frac{e^{\phi(r)}}{4\pi\; e^{[\phi+\Lambda](r)} r^2 sin\,\theta\; \sigma} \Big[e^{\phi(r)} \big(B_{r,\theta} - B_{\theta,r} \big)$$

$$- e^{\Lambda(r)} r^2 sin\,\theta\; \frac{\partial E^{\varphi}}{\partial t} \Big] \Bigg)_{,r} - e^{\phi(r)} \Bigg(\frac{1}{4\pi e^{[\phi+\Lambda](r)} r^2 sin\,\theta\; \sigma} \Big[e^{\phi(r)} \big(B_{\theta,\varphi}$$

$$- B_{\varphi,\theta} \big) - e^{\Lambda(r)} r^2 sin\,\theta\; \frac{\partial E^{r}}{\partial t} \Big] \Bigg)_{,\varphi}$$

sehingga

$$\frac{\partial B^{\theta}}{\partial t} = \frac{1}{e^{\Lambda(r)}r^2 sin\,\theta}\left\{\left(\frac{e^{\phi(r)}}{4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,\sigma}\left[e^{\phi(r)}\left(B_{r,\theta} - B_{\theta,r}\right) - e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial E^{\varphi}}{\partial t}\right]\right)_{,r} - e^{\phi(r)}\left(\frac{1}{4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,\sigma}\left[e^{\phi(r)}\left(B_{\theta,\varphi} - B_{\varphi,\theta}\right) - e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial E^{r}}{\partial t}\right]\right)_{,\varphi}\right\}$$

(C.5)

6. Komponen toroidal

Untuk dinamika medan magnet komponen toroidal, persamaan (C.1) dan (C.2) disubstitusikan ke persamaan (A.42), yaitu

$$e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial B^{\varphi}}{\partial t} = \left(e^{\phi(r)}E_{\theta}\right)_{,r} - e^{\phi(r)}\left(E_r\right)_{,\theta}$$

$$e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial B^{\varphi}}{\partial t} = \left(e^{\phi(r)}\frac{1}{4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,\sigma}\left[e^{\phi(r)}\left(B_{\varphi,r} - B_{r,\varphi}\right) - e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial E^{\theta}}{\partial t}\right]\right)_{,r} - e^{\phi(r)}\left(\frac{1}{4\pi\, e^{[\phi+\Lambda](r)}r^2 sin\,\theta\,\sigma}\left[e^{\phi(r)}\left(B_{\theta,\varphi} - B_{\varphi,\theta}\right) - e^{\Lambda(r)}r^2 sin\,\theta\,\frac{\partial E^{r}}{\partial t}\right]\right)_{,\theta}$$

sehingga

$$\frac{\partial B^{\varphi}}{\partial t} = \frac{1}{e^{\Lambda(r)} r^2 sin\,\theta} \left\{ \left(e^{\phi(r)} \frac{1}{4\pi\, e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,\sigma} \left[e^{\phi(r)} \left(B_{\varphi,r} - B_{r,\varphi} \right) - e^{\Lambda(r)} r^2 sin\,\theta\, \frac{\partial E^{\theta}}{\partial t} \right] \right)_{,r} - e^{\phi(r)} \left(\frac{1}{4\pi\, e^{[\phi+\Lambda](r)} r^2 sin\,\theta\,\sigma} \left[e^{\phi(r)} \left(B_{\theta,\varphi} - B_{\varphi,\theta} \right) - e^{\Lambda(r)} r^2 sin\,\theta\, \frac{\partial E^{r}}{\partial t} \right] \right)_{,\theta} \right\}$$

(C.6)

LAMPIRAN D

PENURUNAN PERSAMAAN GEODESIK

Geodesik adalah lintasan terpendek pada dua titik di sebuah permukaan. Pada koordinat Cartesian, geodesik adalah garis lurus. Pada koordinat lengkung, geodesik adalah garis lengkung.

Di dalam kalkulus variasi (Boas, 1983), prinsip Fermat menyatakan bahwa lintasan cahaya dengan kecepatan v adalah sedemikian rupa sehingga waktu tempuh dari satu titik ke titik lainnya minimum, atau

$$\Delta t = \int_A^B \frac{dv}{v} = \frac{1}{v}\int_A^B (dx^2 + dy^2)^{1/2} \tag{C.1}$$

atau

$$\Delta t . v = \int ds = I. \tag{C.2}$$

Syarat agar I bernilai ekstrem (maksimum atau minimum) harus divariasikan, yakni

$$\delta I = 0 \tag{C.3}$$

sehingga

$$\delta I = \delta \int ds = 0. \tag{C.4}$$

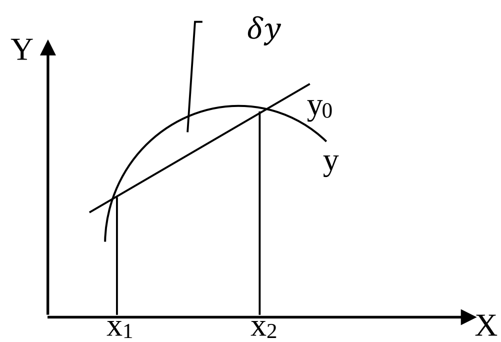
Sementara itu, diberikan elemen garis dari ruang yang hendak kita tinjau adalah

$$ds^2 = g_{\mu\vartheta} dq^{\mu} dq^{\vartheta}. \tag{C.5}$$

Variasi ds^2 adalah δds^2 atau $\delta(ds.ds)$. Sebelum menghitungnya, kita andaikan sebuah fungsi *{x, y(x), y'(x)}* atau *F (x, y', y'')*, di mana *x* tetap. Kita translasikan fungsi tersebut, sehingga

$$\Delta F = F(x, y + \epsilon\mu, y' + \epsilon\mu') - F(x, y, y'). \tag{C.6}$$

Selanjutnya kita definisikan $Y = y(x) + \epsilon\mu(x)$ atau $Y = y(x) + \delta y$ atau $Y = y(x) + (y - y_0)$. Lalu $Y' = y'(x) + \epsilon\mu'(x)$ atau $Y' = y'(x) + \delta dy'$ atau $Y' = y'(x) + (dy' - dy')$. Ide ini berasal dari Euler ketika memandang Gambar 2.3.



Gambar 2.3. Koordinat variasi Euler

Pada grafik *y = y(x)* ini, jika *y = y₀*, akan memenuhi nilai ekstrem baik di x_1 maupun x_2 atau $\delta y = 0$. Pada x tetap, maka $Y = y(x) + \epsilon\mu(x) = y(x)$ dan $Y' = y'(x) + \epsilon\mu'(x) = y'(x)$ (Soedojo, 1995). Kita gunakan deret Taylor untuk $x = a$, $y = b$ dan $y' = c$ (Boas, 1983).

$$F(x, y + \epsilon\mu, y' + \epsilon\mu') = F(a,b,c) + \frac{dF(a,b,c)}{dx}(x-a) + \frac{dF(a,b,c)}{dy}(y-b) + \frac{dF(a,b,c)}{dy'}(y'-c) + \cdots \ldots$$
$$= F(a,b,c) + 0 + \frac{dF(a,b,c)}{dy}(y - y + \epsilon\mu) + \frac{dF(a,b,c)}{dy'}(y' - y' + \epsilon\mu') + \cdots \ldots$$
$$= F(a,b,c) + \frac{dF(a,b,c)}{dy}\epsilon\mu + \frac{dF(a,b,c)}{dy'}\epsilon\mu' + \cdots \ldots$$

(C.7)

sehingga

$$\Delta F = F(x, y + \epsilon\mu, y' + \epsilon\mu') - F(x, y, y')$$
$$= F(a,b,c) + \frac{dF(a,b,c)}{dy}\epsilon\mu + \frac{dF(a,b,c)}{dy'}\epsilon\mu' - F(x,y,y')$$
$$= \frac{dF(a,b,c)}{dy}\epsilon\mu + \frac{dF(a,b,c)}{dy'}\epsilon\mu'$$
$$= \frac{\partial F}{\partial y}\epsilon\mu + \frac{\partial F}{\partial y'}\epsilon\mu'.$$

(C.8)

Ini disebut variasi *F* atau

$$\delta F = \frac{\partial F}{\partial y}\epsilon\mu + \frac{\partial F}{\partial y'}\epsilon\mu'.$$

(C.9)

Kita tahu bahwa $\epsilon\mu = \delta y$, maka variasi *F* dapat ditulis

$$\delta F = \frac{\partial F}{\partial y}\delta y + \frac{\partial F}{\partial y'}\delta y'.$$

(C.10)

Dengan demikian, variasi ds^2 dapat diperoleh

$$\begin{aligned}\delta(ds.ds) &= \frac{\partial(ds.ds)}{\partial y}\delta y + \frac{\partial(ds.ds)}{\partial y'}\delta y' \\ &= \left(ds\frac{\partial ds}{\partial y} + ds\frac{\partial ds}{\partial y}\right)\delta y + \left(ds\frac{\partial ds}{\partial y'} + ds\frac{\partial ds}{\partial y'}\right)\delta y' \\ &= ds\left(\frac{\partial ds}{\partial y}\delta y + \frac{\partial ds}{\partial y}\delta y\right) + ds\left(\frac{\partial ds}{\partial y'}\delta y' + \frac{\partial ds}{\partial y'}\delta y'\right) = ds\delta s + ds\delta s \\ &= 2ds\delta s.\end{aligned}$$

(C.11)

Oleh karena itu, variasi ds^2 adalah

$$2ds\delta ds = \delta g_{\mu\vartheta} dq^{\mu} dq^{\vartheta} + g_{\mu\vartheta}\delta dq^{\mu} dq^{\vartheta} + g_{\mu\vartheta} dq^{\mu}\delta dq^{\vartheta}.$$

(C.12)

Tetapi kita tahu bahwa sifat δ sama dengan sifat *d* dalam kalkulus dan di depan sudah diketahui bahwa

$$\delta dy = dy - dy_0 = dy' - dy$$

(C.13)

atau dengan koordinat umum

$$\delta dq = dq' - dq.$$

(C.14)

Dari kurva di atas, kita dapatkan $\delta y = y - y_0$. Ini juga berlaku untuk $\delta x = x - x_0$. Jika kita pakai koordinat umum, maka $\delta q = q - q_0 = q' - q$ atau $q' = q + \delta q$. Masukkan hasil ini, maka

$$\delta dq = dq' - dq = d(q + \delta q) - dq = d\delta q. \tag{C.15}$$

Sehingga variasi *ds*^{2} dapat ditulis

$$2ds\delta ds = \delta g_{\mu\vartheta} dq^\mu dq^\vartheta + g_{\mu\vartheta} d\delta q^\mu dq^\vartheta + g_{\mu\vartheta} dq^\mu d\delta q^\vartheta. \tag{C.16}$$

Jika persamaan ini dibagi *2ds.ds* lalu dikalikan *ds* dan diintegralkan, maka akan kita dapatkan

$$\delta \int ds = \frac{1}{2} \int \{\delta g_{\mu\vartheta} dq^\mu dq^\vartheta + g_{\mu\vartheta} d\delta q^\mu dq^\vartheta + g_{\mu\vartheta} dq^\mu d\delta q^\vartheta\} ds = 0. \tag{C.17}$$

Dari persamaan (2.74), maka memberikan

$$\frac{1}{2} \int \left\{\delta g_{\mu\vartheta} \frac{dq^\mu}{ds} \frac{dq^\vartheta}{ds} + g_{\mu\vartheta} \frac{d\delta q^\mu}{ds} \frac{dq^\vartheta}{ds} + g_{\mu\vartheta} \frac{dq^\mu}{ds} \frac{d\delta q^\vartheta}{ds}\right\} ds = 0. \tag{C.18}$$

Dengan menggunakan hubungan

$$\delta g_{\mu\vartheta} = \frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}} \delta q^{\gamma}$$

(C.19)

dan

$$\frac{d}{ds}\left(g_{\mu\vartheta}\delta q^{\mu}\frac{dq^{\vartheta}}{ds}\right) - \frac{d}{ds}\left(g_{\mu\vartheta}\frac{dq^{\vartheta}}{ds}\right)\delta q^{\mu}$$

$$= \frac{dg_{\mu\vartheta}}{ds}\delta q^{\mu}\frac{dq^{\vartheta}}{ds} + g_{\mu\vartheta}\frac{d\delta q^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} + g_{\mu\vartheta}\delta q^{\mu}\frac{d^2q^{\vartheta}}{ds^2} - \frac{dg_{\mu\vartheta}}{ds}\delta q^{\mu}\frac{dq^{\vartheta}}{ds}$$

$$+ g_{\mu\vartheta}\delta q^{\mu}\frac{d^2q^{\vartheta}}{ds^2} = g_{\mu\vartheta}\frac{d\delta q^{\mu}}{ds}\frac{dq^{\vartheta}}{ds}.$$

(C.20)

Maka persamaan (2.76) menjadi

$$\frac{1}{2}\int\left\{\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}\delta q^{\gamma}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - \frac{d}{ds}\left(g_{\mu\vartheta}\frac{dq^{\vartheta}}{ds}\right)\delta q^{\mu} - \frac{d}{ds}\left(g_{\mu\vartheta}\frac{dq^{\vartheta}}{ds}\right)\delta q^{\vartheta}\right\}ds = 0$$

(C.21)

atau

$$\frac{1}{2}\int\left\{\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}\delta q^{\gamma}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - \frac{d}{ds}\left(g_{\gamma\vartheta}\frac{dq^{\vartheta}}{ds} + g_{\mu\gamma}\frac{dq^{\mu}}{ds}\right)\delta q^{\gamma}\right\}ds$$

$$= \frac{1}{2}\int\left\{\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - \frac{d}{ds}\left(g_{\gamma\vartheta}\frac{dq^{\vartheta}}{ds} + g_{\mu\gamma}\frac{dq^{\mu}}{ds}\right)\right\}\delta q^{\gamma}ds = 0.$$

(C.22)

δq^{γ} secara umum tidak sama dengan nol, sehingga integrannya harus sama dengan nol. Kita dapat menunjukkan hal ini dengan metode kontradiksi begini: andaikan integrannya tidak sama dengan nol, maka integrannya >0 atau <0, sehingga ruas kiri akan positif, dan ini bertentangan dengan kenyataan bahwa $\delta \int ds = 0$. Jadi integrannya harus sama dengan nol

$$\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - \frac{d}{ds}\left(g_{\gamma\vartheta}\frac{dq^{\vartheta}}{ds} + g_{\mu\gamma}\frac{dq^{\mu}}{ds}\right) = 0. \tag{C.23}$$

Ini disebut persamaan geodesik. Untuk menghubungkan persamaan geodesik dengan Simbol Christoffel, gunakan hubungan

$$\frac{dg_{\mu\gamma}}{ds} = \frac{\partial g_{\mu\gamma}}{\partial q^{\gamma}}\frac{dq^{\gamma}}{ds} \text{ dan } \frac{dg_{\gamma\vartheta}}{ds} = \frac{\partial g_{\gamma\vartheta}}{\partial q^{\mu}}\frac{dq^{\mu}}{ds} \tag{C.24}$$

sehingga memberikan

$$\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - \frac{dg_{\gamma\vartheta}}{ds}\frac{dq^{\vartheta}}{ds} - g_{\gamma\vartheta}\frac{d^2q^{\vartheta}}{ds^2} - \frac{dg_{\mu\gamma}}{ds}\frac{dq^{\mu}}{ds} - g_{\mu\gamma}\frac{d^2q^{\mu}}{ds^2} = 0$$
$$\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - \frac{dg_{\gamma\vartheta}}{ds}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - g_{\gamma\vartheta}\frac{d^2q^{\vartheta}}{ds^2} - \frac{dg_{\mu\gamma}}{dq^{\vartheta}}\frac{dq^{\vartheta}}{ds}\frac{dq^{\mu}}{ds} - g_{\mu\gamma}\frac{d^2q^{\mu}}{ds^2} = 0$$
$$\left(\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}} - \frac{dg_{\mu\gamma}}{dq^{\vartheta}} - \frac{dg_{\gamma\vartheta}}{dq^{\mu}}\right)\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} - g_{\gamma\vartheta}\frac{d^2q^{\vartheta}}{ds^2} - g_{\mu\gamma}\frac{d^2q^{\mu}}{ds^2} = 0$$
$$\left(\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}} - \frac{dg_{\mu\gamma}}{dq^{\vartheta}} - \frac{dg_{\gamma\vartheta}}{dq^{\mu}}\right)\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds} = 2g_{\mu\gamma}\frac{d^2q^{\mu}}{ds^2}. \tag{C.25}$$

Kalikan dengan $g^{\gamma\sigma}$, sehingga

$$g^{\gamma\sigma}\left(\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}-\frac{dg_{\mu\gamma}}{dq^{\vartheta}}-\frac{dg_{\gamma\vartheta}}{dq^{\mu}}\right)\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds}=2g^{\gamma\sigma}g_{\mu\gamma}\frac{d^2q^{\mu}}{ds^2}=2g_{\mu}^{\sigma}\frac{d^2q^{\mu}}{ds^2}=2g_{\sigma}^{\sigma}\frac{d^2q^{\sigma}}{ds^2}$$
$$=2\frac{d^2q^{\sigma}}{ds^2}$$

atau

$$\frac{g^{\gamma\sigma}}{2}\left(\frac{\partial g_{\mu\vartheta}}{\partial q^{\gamma}}-\frac{dg_{\mu\gamma}}{dq^{\vartheta}}-\frac{dg_{\gamma\vartheta}}{dq^{\mu}}\right)\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds}-\frac{d^2q^{\sigma}}{ds^2}=0. \tag{C.26}$$

Dengan menggunakan Simbol Christoffel, maka

$$\Gamma_{\mu\vartheta}^{\sigma}\frac{dq^{\mu}}{ds}\frac{dq^{\vartheta}}{ds}-\frac{d^2q^{\sigma}}{ds^2}=0. \tag{C.27}$$

Ini adalah persamaan geodesik di mana Simbol Christoffel menjadi koefisien kecepatan (Purwanto, 2009).

LAMPIRAN E
PENURUNAN RUANG-WAKTU SCHWARZSCHILD

Diberikan sebuah elemen garis

$$ds^2 = dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2). \tag{D.1}$$

Ini adalah elemen garis koordinat bola tak bergantung waktu. Sedangkan elemen garis koordinat bola bergantung waktu adalah

$$ds^2 = c^2dt^2 - \{dr^2 + r^2(d\theta^2 + sin^2\theta d\varphi^2)\}. \tag{D.2}$$

Ruang-waktu ini disebut sebagai ruang waktu Minkowski, dengan metrik $g_{00} = c^2$, $g_{11} = -1$, $g_{22} = -r^2$, $g_{33} = -r^2 sin^2\theta$. Ruang-waktu ini terbentuk dengan distribusi massa tidak ada ($M = 0$), sehingga bentuk ruang-waktunya datar. Jika ada distribusi massa ($M \neq 0$), maka ruang menjadi melengkung. Oleh karena itu, kita perlu memperumum ruang-waktu pada koordinat simetri bola. Ruang-waktu tersebut berbentuk

$$ds^2 = e^v dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + sin^2\theta d\varphi^2) \tag{D.3}$$

dengan $v = v(r)$, $\lambda = \lambda(r)$ dan tensor metriknya

$$g_{\mu v} = \begin{pmatrix} e^{v} & 0 & 0 & 0 \\ 0 & -e^{\lambda} & 0 & 0 \\ 0 & 0 & -r^{2} & 0 \\ 0 & 0 & 0 & -r^{2}sin^{2}\theta \end{pmatrix}$$

$$g^{\mu v} = \begin{pmatrix} e^{-v} & 0 & 0 & 0 \\ 0 & -e^{-\lambda} & 0 & 0 \\ 0 & 0 & -r^{-2} & 0 \\ 0 & 0 & 0 & -r^{-2}sin^{-2}\theta \end{pmatrix}.$$

(D.4)

Determinannya adalah

$$det g_{\mu v} = g = \begin{pmatrix} e^{v} & 0 & 0 & 0 \\ 0 & -e^{\lambda} & 0 & 0 \\ 0 & 0 & -r^{2} & 0 \\ 0 & 0 & 0 & -r^{2}sin^{2}\theta \end{pmatrix}$$

$$= (e^{v}.[-e^{\lambda}].[-r^{2}].[-r^{2}sin^{2}\theta] + 0 + 0 + 0) - (0 + 0 + 0 + 0) = -e^{v+\lambda}r^{4}sin^{2}\theta.$$

(D.5)

Untuk menentukan nilai e^{v} dan e^{λ}, kita mesti menentukan semua komponen Simbol Christoffel dan tensor kurvatur Ricci. Adapun komponen Simbol Christoffelnya adalah sebagai berikut

$$\Gamma^{\alpha}_{\mu v} = \frac{g^{\alpha\beta}}{2}\left\{\frac{\partial g_{v\beta}}{\partial q^{\mu}} + \frac{\partial g_{\alpha\mu}}{\partial q^{v}} - \frac{\partial g_{\mu v}}{\partial q^{\beta}}\right\} = \frac{g^{\alpha\beta}}{2}\{\partial_{\mu}g_{v\beta} + \partial_{v}g_{\alpha\mu} - \partial_{\beta}g_{\mu v}\}$$

(D.6)

(Hidayat, 2010).

1. Untuk $\mu = v = \alpha$

$$\Gamma^{\mu}_{\mu\mu} = \frac{g^{\mu\beta}}{2}\{\partial_{\mu}g_{\mu\beta} + \partial_{\mu}g_{\mu\mu} - \partial_{\beta}g_{\mu\mu}\}.$$

Tetapi dalam kasus ini, karena komponen tensor metrik non-diagonalnya bernilai nol ($g^{\mu\beta} = 0$), sehingga komponen tensor metrik yang ada harganya hanya yang diagonal saja ($g^{\mu\mu} \neq 0$), maka

$$\Gamma^{\mu}_{\mu\mu} = \frac{g^{\mu\mu}}{2}\{\partial_{\mu} g_{\mu\mu} + \partial_{\mu} g_{\mu\mu} - \partial_{\mu} g_{\mu\mu}\} = \frac{g^{\mu\mu}}{2}\partial_{\mu} g_{\mu\mu} = \frac{1}{2g_{\mu\mu}}\partial_{\mu} g_{\mu\mu}.$$

Dari $\int \frac{du}{u} = ln|u| + C$, maka

$$\Gamma^{\mu}_{\mu\mu} = \frac{1}{2}\frac{\partial_{\mu} g_{\mu\mu}}{g_{\mu\mu}} = \frac{1}{2}\partial_{\mu}\left(\int \frac{\partial_{\mu} g_{\mu\mu}}{g_{\mu\mu}}\right) = \frac{1}{2}\partial_{\mu} ln\ g_{\mu\mu}.$$

Maka untuk Γ^{0}_{00} adalah

$$\Gamma^{0}_{00} = \frac{1}{2}\partial_{0} ln\ g_{00} = \frac{1}{2}\frac{\partial}{\partial t} ln\ e^{v}.$$

Dari $ln\ (e^{y}) = y$, maka

$$\Gamma^{0}_{00} = \frac{1}{2}\frac{\partial v(r)}{\partial t} = 0.$$

Untuk Γ^{1}_{11} adalah

$$\Gamma^{1}_{11} = \frac{1}{2}\partial_{1} ln\ g_{11} = \frac{1}{2}\frac{\partial}{\partial r} ln\ e^{\lambda} = \frac{1}{2}\frac{\partial \lambda(r)}{\partial r} = 0.$$

Untuk Γ^{2}_{22} dan mengingat $\frac{d}{dx} ln\ x = \frac{1}{x}$ adalah

$$\Gamma^2_{22} = \frac{1}{2}\partial_2 ln\ g_{22} = \frac{1}{2}\frac{\partial}{\partial\theta} ln\ r^2 = 0.$$

Untuk Γ^3_{33}

$$\Gamma^3_{33} = \frac{1}{2}\partial_3 ln\ g_{33} = \frac{1}{2}\frac{\partial}{\partial\varphi}(r^2 sin^2\theta) = 0.$$

2. Untuk $\mu = \nu \neq \alpha$ di mana $\beta = \alpha$

$$\Gamma^\alpha_{\mu\mu} = \frac{g^{\alpha\alpha}}{2}\{\partial_\mu g_{\mu\alpha} + \partial_\mu g_{\alpha\mu} - \partial_\alpha g_{\mu\mu}\} = \frac{g^{\alpha\alpha}}{2}\{2\partial_\mu g_{\mu\alpha} - \partial_\alpha g_{\mu\mu}\}.$$

Karena $\mu \neq \alpha$, sedangkan kita mencari metrik diagonalnya, maka $2\partial_\mu g_{\mu\alpha} = 2\partial_\mu 0 = 0$. Sehingga

$$\Gamma^\alpha_{\mu\mu} = -\frac{g^{\alpha\alpha}}{2}\partial_\alpha g_{\mu\mu}.$$

Untuk Γ^0_{11}

$$\Gamma^0_{11} = -\frac{g^{00}}{2}\partial_0 g_{11} = -\frac{1}{2}e^{-\nu}\frac{\partial}{\partial t}(-e^{-\lambda}) = 0.$$

Untuk Γ^0_{22}

$$\Gamma^0_{22} = -\frac{g^{00}}{2}\partial_0 g_{22} = -\frac{1}{2}e^{-\lambda}\frac{\partial}{\partial t}(-r^2) = 0.$$

Untuk Γ^0_{33}

$$\Gamma^0_{33} = -\frac{g^{00}}{2}\partial_0 g_{33} = -\frac{1}{2}e^{-\lambda}\frac{\partial}{\partial t}(-r^2 sin^2\theta) = 0.$$

Untuk Γ^1_{00} dan mengingat $\frac{de^u}{dx} = e^u\frac{du}{dx}$

$$\Gamma^1_{00} = -\frac{g^{11}}{2}\partial_1 g_{00} = \frac{e^{-\lambda}}{2}\frac{\partial e^v}{\partial r} = \frac{e^{-\lambda}}{2}e^v\frac{\partial v}{\partial r} = \frac{1}{2}(e^{-\lambda}e^v)v' = \frac{v'}{2}e^{-\lambda+v}.$$

Untuk Γ^1_{11}

$$\Gamma^1_{11} = -\frac{g^{11}}{2}\partial_1 g_{11} = \frac{e^{-\lambda}}{2}\frac{\partial(e^{-\lambda})}{\partial r} = \frac{e^{-\lambda}}{2}e^{-\lambda}\frac{\partial\lambda}{\partial r} = \frac{1}{2}\frac{\partial\lambda}{\partial r}.$$

Untuk Γ^1_{22}

$$\Gamma^1_{22} = -\frac{g^{11}}{2}\partial_1 g_{22} = \frac{e^{-\lambda}}{2}\frac{\partial(r^2)}{\partial r} = \frac{e^{-\lambda}}{2}\frac{\partial r^2}{\partial r} = re^{-\lambda}.$$

Untuk Γ^1_{33}

$$\Gamma^1_{33} = -\frac{g^{11}}{2}\partial_1 g_{33} = \frac{e^{-\lambda}}{2}\frac{\partial(r^2 sin^2\theta)}{\partial r} = \frac{e^{-\lambda}}{2}\left(\frac{\partial r^2}{\partial r}sin^2\theta + r^2\frac{\partial sin^2\theta}{\partial r}\right)$$
$$= \frac{e^{-\lambda}}{2}(2rsin^2\theta + 0) = r\,sin^2\theta\,e^{-\lambda}.$$

Untuk Γ^2_{00}

$$\Gamma^2_{00} = -\frac{g^{22}}{2}\partial_2 g_{00} = \frac{(r^{-2})}{2}\frac{\partial e^v}{\partial\theta} = 0.$$

Untuk Γ_{11}^{2}

$$\Gamma_{11}^{2} = -\frac{g^{22}}{2}\partial_{2}g_{11} = \frac{(r^{-2})}{2}\frac{\partial e^{\lambda}}{\partial\theta} = 0.$$

Untuk Γ_{22}^{2}

$$\Gamma_{22}^{2} = -\frac{g^{22}}{2}\partial_{2}g_{22} = \frac{(r^{-2})}{2}\frac{\partial r^{-2}}{\partial\theta} = 0.$$

Untuk Γ_{33}^{2}

$$\Gamma_{33}^{2} = -\frac{g^{22}}{2}\partial_{2}g_{33} = \frac{(r^{-2})}{2}\frac{\partial(r^{2}sin^{2}\theta)}{\partial\theta} = \frac{1}{2r^{2}}\left(\frac{\partial r^{2}}{\partial\theta}sin^{2}\theta + r^{2}\frac{\partial sin^{2}\theta}{\partial\theta}\right)$$
$$= \frac{1}{2r^{2}}\left(0 + r^{2}\frac{\partial sin^{2}\theta}{\partial\theta}\right) = \frac{1}{2}\frac{\partial sin^{2}\theta}{\partial\theta}.$$

Misalkan $y = u^{2}$dengan $u = sin\theta$, maka $\frac{dy}{du} = 2u$ dan $\frac{du}{d\theta} = cos\ \theta$, sehingga $\frac{dy}{d\theta} = \frac{dy}{du}\frac{du}{d\theta} = 2u\ cos\ \theta = 2sin\ \theta cos\ \theta$. Dengan begitu

$$\Gamma_{33}^{2} = \frac{1}{2}2sin\ \theta cos\ \theta = sin\ \theta cos\ \theta.$$

Untuk Γ_{00}^{3}

$$\Gamma_{00}^{3} = -\frac{g^{33}}{2}\partial_{3}g_{00} = \frac{(r^{-2})}{2}sin^{2}\theta\frac{\partial(e^{v})}{\partial\varphi} = 0.$$

Komponen $\Gamma^3_{11} = \Gamma^3_{22} = \Gamma^3_{33} = \Gamma^3_{00} = 0$, karena $g_{\mu\mu}$ bukan fungsi φ.

3. Untuk $v = \alpha \neq \mu$ di mana $\beta = v$

$$\Gamma^v_{\mu v} = \frac{g^{vv}}{2}\left\{\partial_\mu g_{vv} + \partial_v g_{\alpha\mu} - \partial_v g_{\mu v}\right\} = \frac{g^{vv}}{2}\partial_\mu g_{vv} = \frac{1}{2}\frac{\partial_\mu g^{vv}}{g_{vv}}$$
$$= \frac{1}{2}\partial_\mu\left(\int \frac{\partial_\mu g^{vv}}{g_{vv}}\right) = \frac{1}{2}\partial_\mu \, ln\, g_{vv}.$$

Untuk Γ^0_{10} dan mengingat *ln (e*^{y}*) = y*

$$\Gamma^0_{10} = \frac{1}{2}\partial_1 \, ln\, g_{00} = \frac{1}{2}\frac{\partial(ln\, e^v)}{\partial r} = \frac{1}{2}\frac{\partial v}{\partial r} = \frac{v'}{2}.$$

Untuk Γ^0_{20}

$$\Gamma^0_{20} = \frac{1}{2}\partial_2 \, ln\, g_{00} = \frac{1}{2}\frac{\partial(ln\, e^v)}{\partial \theta} = 0 .$$

Untuk Γ^0_{30}

$$\Gamma^0_{30} = \frac{1}{2}\partial_3 \, ln\, g_{00} = \frac{1}{2}\frac{\partial(ln\, e^v)}{\partial \varphi} = 0 .$$

Untuk Γ^1_{01}

$$\Gamma^1_{01} = \frac{1}{2}\partial_0 \, ln\, g_{11} = \frac{1}{2}\frac{\partial(ln\, -e^\lambda)}{\partial t} = 0 .$$

Dengan cara yang sama, $\Gamma^1_{21} = \Gamma^1_{31} = 0$.

Untuk Γ^2_{02}

$$\Gamma^2_{02} = \frac{1}{2}\partial_0\ ln\ g_{22} = \frac{1}{2}\frac{\partial(ln -r^2)}{\partial t} = 0\ .$$

Untuk Γ^2_{12} serta mengingat $u = r^2$dan $\frac{d}{dx} ln\ u = \frac{1}{u}\frac{d}{dx} u$

$$\Gamma^2_{12} = \frac{1}{2}\partial_1\ ln\ g_{22} = \frac{1}{2}\frac{\partial(ln -r^2)}{\partial r} = \frac{1}{2}\frac{1}{r^2}\frac{\partial r^2}{\partial r} = \frac{2}{2}\frac{r}{r^2} = \frac{1}{r}\ .$$

Untuk Γ^2_{32}

$$\Gamma^2_{32} = \frac{1}{2}\partial_3\ ln\ g_{22} = \frac{1}{2}\frac{\partial(ln -r^2)}{\partial \varphi} = 0\ .$$

Untuk Γ^3_{03}

$$\Gamma^3_{03} = \frac{1}{2}\partial_0\ ln\ g_{33} = \frac{1}{2}\frac{\partial(ln\ r^2 sin^2\theta)}{\partial t} = 0\ .$$

Untuk Γ^3_{13}

$$\Gamma^3_{13} = \frac{1}{2}\partial_1\ ln\ g_{33} = \frac{1}{2}\frac{\partial(ln\ r^2 sin^2\theta)}{\partial t} = \frac{1}{2}\left(\frac{\partial}{\partial r} ln\ r^2 + \frac{\partial}{\partial r} ln\ sin^2\theta\right)$$
$$= \frac{1}{r} + 0 = \frac{1}{r}\ .$$

Untuk Γ^3_{23}

$$\Gamma^3_{23} = \frac{1}{2}\partial_2\, ln\, g_{33} = \frac{1}{2}\frac{\partial(ln\, r^2 + ln\, sin^2\theta)}{\partial\theta} = \frac{1}{2}\frac{\partial(ln\, sin^2\theta)}{\partial\theta}$$

dengan memisalkan $y = u^2$ dan $u = sin^2\theta$, maka $\frac{dy}{du} = 2u$, $\frac{du}{d\theta} = cos\ \theta$, serta $\frac{dy}{du}\frac{du}{d\theta} = 2sin\ \theta\ cos\ \theta$, sehingga

$$\Gamma^3_{23} = \frac{1}{2}\frac{\partial(ln\, u^2)}{\partial\theta} = \frac{1}{2}\frac{1}{u^2}\frac{du^2}{d\theta} = \frac{1}{2}\frac{1}{sin^2\theta}\frac{d}{d\theta}sin^2\theta = \frac{1}{2}\frac{1}{sin^2\theta}2sin\ \theta\ cos\ \theta = \frac{cos\ \theta}{sin\ \theta}$$
$$= cot\ \theta.$$

4. Untuk $\mu \neq v \neq \alpha$ di mana $\beta = \alpha$

$$\Gamma^\alpha_{\mu v} = \frac{g^{\alpha\alpha}}{2}\{\partial_\mu g_{v\alpha} + \partial_v g_{\alpha\mu} - \partial_\alpha g_{\mu v}\} = 0.$$

Jadi, ada 9 komponen Simbol Christoffel tak nol, yaitu

$$\Gamma^0_{10} = \frac{v\prime}{2},\ \Gamma^1_{00} = \frac{v\prime}{2}e^{-\lambda+v},\ \Gamma^2_{12} = \frac{1}{r},\ \Gamma^3_{13} = \frac{1}{r},\ \Gamma^1_{11} = \frac{1}{2}\frac{\partial\lambda}{\partial r},\ \Gamma^2_{33} = sin\theta cos\theta$$
$$\Gamma^3_{23} = cot\theta,\ \Gamma^1_{22} = re^{-\lambda},\ \Gamma^1_{33} = rsin^2\theta e^{-\lambda}. \quad \text{(D.7)}$$

Nilai-nilai tersebut, kita masukkan ke dalam tensor Ricci

$$R_{\mu\vartheta} = \partial_\alpha\Gamma^\alpha_{\mu\vartheta} - \partial_\mu\Gamma^\alpha_{\alpha\vartheta} + \Gamma^\beta_{\mu\vartheta}\Gamma^\alpha_{\alpha\beta} - \Gamma^\beta_{\alpha\vartheta}\Gamma^\alpha_{\mu\beta}. \quad \text{(D.8)}$$

Semua suku non-diagonalnya (μv) lenyap, sementara suku diagonalnya ($\mu\mu$) adalah

$$R_{\mu\mu} = \partial_\alpha \Gamma^\alpha_{\mu\mu} - \partial_\mu \Gamma^\alpha_{\alpha\mu} + \Gamma^\beta_{\mu\mu}\Gamma^\alpha_{\alpha\beta} - \Gamma^\beta_{\alpha\mu}\Gamma^\alpha_{\mu\beta}. \tag{D.9}$$

5. Untuk R_{00}

$$\begin{aligned}
R_{00} &= \partial_0 \Gamma^0_{00} + \partial_1 \Gamma^1_{00} + \partial_2 \Gamma^2_{00} + \partial_3 \Gamma^3_{00} \\
&\quad -\partial_0 \Gamma^0_{00} - \partial_0 \Gamma^1_{01} - \partial_0 \Gamma^2_{02} - \partial_0 \Gamma^3_{03} \\
&\quad +\Gamma^0_{00}\Gamma^0_{00} + \Gamma^0_{00}\Gamma^1_{01} + \Gamma^0_{00}\Gamma^2_{02} + \Gamma^0_{00}\Gamma^3_{03} \\
&\quad +\Gamma^1_{00}\Gamma^0_{10} + \Gamma^1_{00}\Gamma^1_{11} + \Gamma^1_{00}\Gamma^2_{12} + \Gamma^1_{00}\Gamma^3_{13} \\
&\quad +\Gamma^2_{00}\Gamma^0_{20} + \Gamma^2_{00}\Gamma^1_{21} + \Gamma^2_{00}\Gamma^2_{22} + \Gamma^2_{00}\Gamma^3_{23} \\
&\quad +\Gamma^3_{00}\Gamma^0_{30} + \Gamma^3_{00}\Gamma^1_{31} + \Gamma^3_{00}\Gamma^2_{32} + \Gamma^3_{00}\Gamma^3_{33} \\
&\quad -\Gamma^0_{00}\Gamma^0_{00} - \Gamma^0_{01}\Gamma^1_{00} - \Gamma^0_{02}\Gamma^2_{00} - \Gamma^0_{03}\Gamma^3_{00} \\
&\quad -\Gamma^1_{00}\Gamma^0_{10} - \Gamma^1_{01}\Gamma^1_{10} - \Gamma^1_{02}\Gamma^2_{10} - \Gamma^1_{03}\Gamma^3_{10} \\
&\quad -\Gamma^2_{00}\Gamma^0_{20} - \Gamma^2_{01}\Gamma^1_{20} - \Gamma^2_{02}\Gamma^2_{20} - \Gamma^2_{03}\Gamma^3_{20} \\
&\quad -\Gamma^3_{00}\Gamma^0_{30} - \Gamma^3_{01}\Gamma^1_{30} - \Gamma^3_{02}\Gamma^2_{30} - \Gamma^3_{03}\Gamma^3_{30} \\
&= 0 + \partial_1 \Gamma^1_{00} + 0 + 0 \\
&\quad -0 - 0 - 0 - 0 \\
&\quad +0 + 0 + 0 + 0 \\
&\quad +\Gamma^1_{00}\Gamma^0_{10} + \Gamma^1_{00}\Gamma^1_{11} + \Gamma^1_{00}\Gamma^2_{12} + \Gamma^1_{00}\Gamma^3_{13} \\
&\quad +0 + 0 + 0 + 0 \\
&\quad +0 + 0 + 0 + 0 \\
&\quad -0 - \Gamma^0_{01}\Gamma^1_{00} - 0 - 0 \\
&\quad -\Gamma^1_{00}\Gamma^0_{10} - 0 - 0 - 0 \\
&\quad -0 - 0 - 0 - 0 \\
&\quad -0 - 0 - 0 - 0 \\
&= \partial_1 \Gamma^1_{00} + \Gamma^1_{00}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}) - \Gamma^0_{01}\Gamma^1_{00} - \Gamma^1_{00}\Gamma^0_{10} \\
&= \frac{\partial}{\partial r}\left(\frac{1}{2}\frac{\partial v}{\partial r}e^{v-\lambda}\right) + \frac{1}{2}\frac{\partial v}{\partial r}e^{v-\lambda}\left(\frac{1}{2}\frac{\partial v}{\partial r} + \frac{1}{2}\frac{\partial \lambda}{\partial r} + \frac{2}{r}\right) - 2\,\frac{1}{2}\frac{\partial v}{\partial r}e^{v-\lambda}\,\frac{1}{2}\frac{\partial v}{\partial r}
\end{aligned}$$

$$= \frac{1}{2}\left(\frac{\partial v'}{\partial r}e^{v-\lambda} + v'\frac{\partial e^{v-\lambda}}{\partial r}\right) + \frac{v'}{2}e^{v-\lambda}\left(\frac{v'+\lambda'}{2} + \frac{2}{r}\right) - \frac{v'^2}{2}e^{v-\lambda}.$$

Mengingat $\frac{de^u}{dx} = e^u\frac{du}{dx}$, maka

$$R_{00} = \frac{v''}{2}e^{v-\lambda} + \frac{v'}{2}e^{v-\lambda}\frac{\partial(v-\lambda)}{\partial r} + \frac{v'}{2}e^{v-\lambda}\left(\frac{2}{r} + \frac{v'+\lambda'}{2}\right) - \frac{v'^2}{2}e^{v-\lambda}$$

$$= e^{v-\lambda}\left(\frac{v''}{2} + \frac{v'}{2}(v-\lambda)\right) + e^{v-\lambda}\left(\frac{v'}{r} + \frac{v'^2}{4} + \frac{v'\lambda'}{4}\right) - \frac{v'^2}{2}e^{v-\lambda}$$

$$= e^{v-\lambda}\left(\frac{v''}{2} + \frac{v'}{r} + \frac{v'^2}{2} - \frac{v'\lambda'}{2} + \frac{v'^2}{4} + \frac{v'\lambda'}{4} - \frac{v'^2}{2}\right)$$

$$= e^{v-\lambda}\left\{\frac{v''}{2} + \frac{v'}{r} - \frac{v'\lambda'}{4} + \frac{v'^2}{4}\right\}.$$

6. Untuk R_{11}

$$R_{11} = \partial_0\Gamma^0_{11} + \partial_1\Gamma^1_{11} + \partial_2\Gamma^2_{11} + \partial_3\Gamma^3_{11}$$
$$-\partial_1\Gamma^0_{10} - \partial_1\Gamma^1_{11} - \partial_1\Gamma^2_{12} - \partial_1\Gamma^3_{13}$$
$$+\Gamma^0_{11}\Gamma^0_{00} + \Gamma^0_{11}\Gamma^1_{01} + \Gamma^0_{11}\Gamma^2_{02} + \Gamma^0_{11}\Gamma^3_{03}$$
$$+\Gamma^1_{11}\Gamma^0_{10} + \Gamma^1_{11}\Gamma^1_{11} + \Gamma^1_{11}\Gamma^2_{12} + \Gamma^1_{11}\Gamma^3_{13}$$
$$+\Gamma^2_{11}\Gamma^0_{20} + \Gamma^2_{11}\Gamma^1_{21} + \Gamma^2_{11}\Gamma^2_{22} + \Gamma^2_{11}\Gamma^3_{23}$$
$$+\Gamma^3_{11}\Gamma^0_{30} + \Gamma^3_{11}\Gamma^1_{31} + \Gamma^3_{11}\Gamma^2_{32} + \Gamma^3_{11}\Gamma^3_{33}$$
$$-\Gamma^0_{10}\Gamma^0_{01} - \Gamma^0_{11}\Gamma^1_{01} - \Gamma^0_{12}\Gamma^2_{01} - \Gamma^0_{13}\Gamma^3_{01}$$
$$-\Gamma^1_{10}\Gamma^0_{11} - \Gamma^1_{11}\Gamma^1_{11} - \Gamma^1_{12}\Gamma^2_{11} - \Gamma^1_{13}\Gamma^3_{11}$$
$$-\Gamma^2_{10}\Gamma^0_{21} - \Gamma^2_{11}\Gamma^1_{21} - \Gamma^2_{12}\Gamma^2_{21} - \Gamma^2_{13}\Gamma^3_{21}$$
$$-\Gamma^3_{10}\Gamma^0_{31} - \Gamma^3_{11}\Gamma^1_{31} - \Gamma^3_{12}\Gamma^2_{31} - \Gamma^3_{13}\Gamma^3_{31}$$
$$= 0 + \partial_1\Gamma^1_{11} + 0 + 0$$
$$-\partial_1\Gamma^0_{10} - \partial_1\Gamma^1_{11} - \partial_1\Gamma^2_{12} - \partial_1\Gamma^3_{13}$$
$$+0 + 0 + 0 + 0$$

$$+\Gamma^1_{11}\Gamma^0_{10} + \Gamma^1_{11}\Gamma^1_{11} + \Gamma^1_{11}\Gamma^2_{12} + \Gamma^1_{11}\Gamma^3_{13}$$
$$+0 + 0 + 0 + 0$$
$$+0 + 0 + 0 + 0$$
$$-\Gamma^0_{10}\Gamma^0_{01} - 0 - 0 - 0$$
$$-0 - \Gamma^1_{11}\Gamma^1_{11} - 0 - 0$$
$$-0 - 0 - \Gamma^2_{12}\Gamma^2_{21} - 0$$
$$-0 - 0 - 0 - \Gamma^3_{13}\Gamma^3_{31}$$
$$= \partial_1\Gamma^1_{11} - \partial_1(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}) + \Gamma^1_{11}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}) - (\Gamma^0_{10}\Gamma^0_{01} + \Gamma^1_{11}\Gamma^1_{11} + \Gamma^2_{12}\Gamma^2_{21} + \Gamma^3_{13}\Gamma^3_{31})$$
$$= \frac{\partial}{\partial r}\left(\frac{1}{2}\frac{\partial\lambda}{\partial r}\right) - \frac{\partial}{\partial r}\left(\frac{1}{2}\frac{\partial v}{\partial r} + \frac{1}{2}\frac{\partial\lambda}{\partial r} + \frac{2}{r}\right) + \frac{1}{2}\frac{\partial v}{\partial r}\left(\frac{1}{2}\frac{\partial v}{\partial r} + \frac{1}{2}\frac{\partial\lambda}{\partial r} + \frac{2}{r}\right) + \left(\frac{1}{2}\frac{\partial v}{\partial r}\right)^2 + \left(\frac{1}{2}\frac{\partial\lambda}{\partial r}\right)^2 + \left(\frac{1}{r}\right)^2 + \left(\frac{1}{r}\right)^2$$
$$= \frac{1}{2}\lambda'' - \left(\frac{v''}{2} + \frac{\lambda''}{2} + \frac{2}{r^2}\right) + \left(\frac{v^2}{4} + \frac{v'\lambda'}{4} + \frac{v'}{r}\right) - \frac{v^2}{2} - \frac{2}{r^2}$$
$$= -\frac{v''}{2} - \frac{v'^2}{4} + \frac{v'\lambda'}{4} + \frac{v'}{r}$$
$$R_{11} = -\frac{v''}{2} + \frac{\lambda'}{r} - \frac{v'\lambda'}{4} + \frac{v'^2}{4}.$$

7. Untuk R_{22}

$$R_{22} = \partial_0\Gamma^0_{22} + \partial_1\Gamma^1_{22} + \partial_2\Gamma^2_{22} + \partial_3\Gamma^3_{22}$$
$$-\partial_2\Gamma^0_{20} - \partial_2\Gamma^1_{21} - \partial_2\Gamma^2_{22} - \partial_2\Gamma^3_{23}$$
$$+\Gamma^0_{22}\Gamma^0_{00} + \Gamma^0_{22}\Gamma^1_{01} + \Gamma^0_{22}\Gamma^2_{02} + \Gamma^0_{22}\Gamma^3_{03}$$
$$+\Gamma^1_{22}\Gamma^0_{10} + \Gamma^1_{22}\Gamma^1_{11} + \Gamma^1_{22}\Gamma^2_{12} + \Gamma^1_{22}\Gamma^3_{13}$$
$$+\Gamma^2_{22}\Gamma^0_{20} + \Gamma^2_{22}\Gamma^1_{21} + \Gamma^2_{22}\Gamma^2_{22} + \Gamma^2_{22}\Gamma^3_{23}$$
$$+\Gamma^3_{22}\Gamma^0_{30} + \Gamma^3_{22}\Gamma^1_{31} + \Gamma^3_{22}\Gamma^2_{32} + \Gamma^3_{22}\Gamma^3_{33}$$
$$-\Gamma^0_{20}\Gamma^0_{02} - \Gamma^0_{21}\Gamma^1_{02} - \Gamma^0_{22}\Gamma^2_{02} - \Gamma^0_{23}\Gamma^3_{02}$$
$$-\Gamma^1_{20}\Gamma^0_{12} - \Gamma^1_{21}\Gamma^1_{12} - \Gamma^1_{22}\Gamma^2_{12} - \Gamma^1_{23}\Gamma^3_{12}$$

$$-\Gamma^2_{20}\Gamma^0_{22} - \Gamma^2_{21}\Gamma^1_{22} - \Gamma^2_{22}\Gamma^2_{22} - \Gamma^2_{23}\Gamma^3_{22}$$

$$-\Gamma^3_{20}\Gamma^0_{32} - \Gamma^3_{21}\Gamma^1_{32} - \Gamma^3_{22}\Gamma^2_{32} - \Gamma^3_{23}\Gamma^3_{32}$$

$$= 0 + \partial_1\Gamma^1_{22} + 0 + 0$$

$$-0 - 0 - 0 - \partial_2\Gamma^3_{23}$$

$$+0 + 0 + 0 + 0$$

$$+\Gamma^1_{22}\Gamma^0_{10} + \Gamma^1_{22}\Gamma^1_{11} + \Gamma^1_{22}\Gamma^2_{12} + \Gamma^1_{22}\Gamma^3_{13}$$

$$+0 + 0 + 0 + 0$$

$$+0 + 0 + 0 + 0$$

$$-0 - 0 - 0 - 0$$

$$-0 - 0 - \Gamma^1_{22}\Gamma^2_{12} - 0$$

$$-0 - \Gamma^2_{21}\Gamma^1_{22} - 0 - 0$$

$$-0 - 0 - 0 - \Gamma^3_{23}\Gamma^3_{32}$$

$$= \partial_1\Gamma^1_{22} - \partial_2\Gamma^3_{23} + \Gamma^1_{22}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}) - (\Gamma^1_{22}\Gamma^2_{12} + \Gamma^2_{21}\Gamma^1_{22} + \Gamma^3_{23}\Gamma^3_{32})$$

$$= -\frac{\partial}{\partial r}\left(re^{-\lambda}\right)\frac{\partial}{\partial\theta}\frac{\cos\theta}{\sin\theta} - re^{-\lambda}\left(\frac{1}{2}\frac{\partial v}{\partial r} + \frac{1}{2}\frac{\partial\lambda}{\partial r} + \frac{2}{r}\right) - 2\left(-re^{-\lambda}\frac{1}{r}\right) - \cot^2\theta$$

$$= \left(-\frac{\partial e^{-\lambda}}{\partial r}r - e^{-\lambda}\frac{\partial r}{\partial r}\right) - \left(\frac{\sin\theta\,\frac{\partial(\cos\theta)}{\partial\theta} - \cos\theta\,\frac{\partial(\sin\theta)}{\partial\theta}}{\sin^2\theta}\right)$$

$$- re^{-\lambda}\left(\frac{2}{r} + \frac{\lambda' + v'}{2}\right) + 2e^{-\lambda} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{\partial\lambda}{\partial r}re^{-\lambda} - e^{-\lambda} + \frac{\sin\theta\sin\theta - \cos\theta\cos\theta}{\sin^2\theta} - re^{-\lambda}\left(\frac{2}{r} + \frac{\lambda' + v'}{2}\right) + 2e^{-\lambda} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= -e^{-\lambda} + \lambda' re^{-\lambda} + \frac{1}{\sin^2\theta} - re^{-\lambda}\left(\frac{2}{r} + \frac{\lambda' + v'}{2}\right) + 2e^{-\lambda} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= -e^{-\lambda} + r\left(\lambda' - \frac{2}{r} - \frac{\lambda' + v'}{2} + \frac{2}{r}\right)e^{-\lambda} + \left(\frac{1 - \cos^2\theta}{\sin^2\theta}\right)$$

$$= -e^{-\lambda} + r\left(\frac{2\lambda' - \lambda' - v'}{2}\right)e^{-\lambda} + \frac{\sin^2\theta}{\sin^2\theta}$$

$$R_{22} = -e^{-\lambda}\left\{\frac{r(v' - \lambda')}{2} + 1\right\} + 1.$$

8. Untuk R_{33}

$$\begin{aligned}
R_{33} &= \partial_0\Gamma^0_{33} + \partial_1\Gamma^1_{33} + \partial_2\Gamma^2_{33} + \partial_3\Gamma^3_{33} \\
&-\partial_3\Gamma^0_{30} - \partial_3\Gamma^1_{31} - \partial_3\Gamma^2_{32} - \partial_3\Gamma^3_{33} \\
&+\Gamma^0_{33}\Gamma^0_{00} + \Gamma^0_{33}\Gamma^1_{01} + \Gamma^0_{33}\Gamma^2_{02} + \Gamma^0_{33}\Gamma^3_{03} \\
&+\Gamma^1_{33}\Gamma^0_{10} + \Gamma^1_{33}\Gamma^1_{11} + \Gamma^1_{33}\Gamma^2_{12} + \Gamma^1_{33}\Gamma^3_{13} \\
&+\Gamma^2_{33}\Gamma^0_{20} + \Gamma^2_{33}\Gamma^1_{21} + \Gamma^2_{33}\Gamma^2_{22} + \Gamma^2_{33}\Gamma^3_{23} \\
&+\Gamma^3_{33}\Gamma^0_{30} + \Gamma^3_{33}\Gamma^1_{31} + \Gamma^3_{33}\Gamma^2_{32} + \Gamma^3_{33}\Gamma^3_{33} \\
&-\Gamma^0_{30}\Gamma^0_{03} - \Gamma^0_{31}\Gamma^1_{03} - \Gamma^0_{32}\Gamma^2_{03} - \Gamma^0_{33}\Gamma^3_{03} \\
&-\Gamma^1_{30}\Gamma^0_{13} - \Gamma^1_{31}\Gamma^1_{13} - \Gamma^1_{32}\Gamma^2_{13} - \Gamma^1_{33}\Gamma^3_{13} \\
&-\Gamma^2_{30}\Gamma^0_{23} - \Gamma^2_{31}\Gamma^1_{23} - \Gamma^2_{32}\Gamma^2_{23} - \Gamma^2_{33}\Gamma^3_{23} \\
&-\Gamma^3_{30}\Gamma^0_{33} - \Gamma^3_{31}\Gamma^1_{33} - \Gamma^3_{32}\Gamma^2_{33} - \Gamma^3_{33}\Gamma^3_{33} \\
&= 0 + \partial_1\Gamma^1_{33} + \partial_2\Gamma^2_{33} + 0 \\
&-0 - 0 - 0 - 0 \\
&+0 + 0 + 0 + 0 \\
&+\Gamma^1_{33}\Gamma^0_{10} + \Gamma^1_{33}\Gamma^1_{11} + \Gamma^1_{33}\Gamma^2_{12} + \Gamma^1_{33}\Gamma^3_{13} \\
&+0 + 0 + 0 + \Gamma^2_{33}\Gamma^3_{23} \\
&+0 + 0 + 0 + 0 \\
&-0 - 0 - 0 - 0 \\
&-0 - 0 - 0 - \Gamma^1_{33}\Gamma^3_{13} \\
&-0 - 0 - 0 - \Gamma^2_{33}\Gamma^3_{23} \\
&-0 - \Gamma^3_{31}\Gamma^1_{33} - \Gamma^3_{32}\Gamma^2_{33} - 0
\end{aligned}$$

$$= (\partial_1\Gamma^1_{33} + \partial_2\Gamma^2_{33}) + \Gamma^1_{33}(\Gamma^0_{10} + \Gamma^1_{11} + \Gamma^2_{12} + \Gamma^3_{13}) + \Gamma^2_{33}\Gamma^3_{23} - \Gamma^1_{33}\Gamma^3_{13} - \Gamma^2_{33}\Gamma^3_{23} - (\Gamma^3_{31}\Gamma^1_{33} + \Gamma^3_{32}\Gamma^2_{33})$$

$$= \frac{\partial}{\partial r}\left(-rsin^2\theta e^{-\lambda}\right) + \frac{\partial}{\partial \theta}(-sin\,\theta cos\,\theta) - rsin^2\theta e^{-\lambda}\left(\frac{1}{2}\frac{\partial v}{\partial r} + \frac{1}{2}\frac{\partial \lambda}{\partial r} + \frac{2}{r}\right)$$

$$+ \frac{2rsin^2\theta e^{-\lambda}}{r} + sin\theta\, cos\theta\, cot\theta$$

$$= \left(-\frac{\partial r}{\partial r}sin^2\theta e^{-\lambda} - \frac{\partial e^{-\lambda}}{\partial r}\right) + \left(\frac{\partial(-sin\,\theta)}{\partial \theta}cos\,\theta - sin\,\theta\frac{\partial(cos\,\theta)}{\partial \theta}\right)$$

$$- sin^2\theta e^{-\lambda}\left(r\frac{v' + \lambda'}{2} + 2\right) + 2\,sin^2\theta e^{-\lambda} + \frac{sin\,\theta\,cos\,\theta\,cos\theta}{sin\,\theta}$$

$$= -sin^2\theta e^{-\lambda} + r\,sin^2\theta\,\lambda' e^{-\lambda} - cos^2\theta + sin^2\theta - sin^2\theta e^{-\lambda}\left(2 + r\frac{v' + \lambda'}{2} - 2\right)$$

$$+ cos^2\theta$$

$$= -sin^2\theta e^{-\lambda}\left(1 - r\lambda' + \frac{v' + \lambda'}{2}\right) + sin^2\theta$$

$$= -sin^2\theta e^{-\lambda}\left(1 - \frac{2r\lambda'}{2} + \frac{r\lambda'}{2} + \frac{rv'}{2}\right) + sin^2\theta$$

$$R_{33} = -sin^2\theta e^{-\lambda}\left\{\frac{r}{2}(v' - \lambda') + 1\right\} + sin^2\theta = sin^2\theta R_{22}.$$

Kompoen Ricci di atas akan jadi tiga persamaan diferensial dengan suku eksponensial pada $R_{00}e^{0-0} = 1$. Sementara itu, R_{33} tidak diperlukan, karena hanya dibedakan oleh suku $sin^2\theta$ dengan R_{22}. Ketiga persamaan tersebut adalah

$$\frac{v''}{2} + \frac{v'}{r} - \frac{v'\lambda'}{4} + \frac{v'^2}{4} = 0$$

$$\frac{v''}{2} - \frac{\lambda'}{r} - \frac{v'\lambda'}{4} + \frac{v'^2}{4} = 0$$

$$e^{-\lambda}\left\{\frac{r(v' - \lambda')}{2} + 1\right\} - 1 = 0. \tag{D.10}$$

Selisih dua persamaan pertama (2.105) memberikan

$$\frac{v' + \lambda'}{r} = 0$$

(D.11)

atau

$$v' + \lambda' = 0$$

(D.12)

sehingga

$$v' = -\lambda'.$$

(D.13)

Kita integralkan

$$\int \left(\frac{\partial v}{\partial r} + \frac{\partial \lambda}{\partial r}\right) dr = 0$$

$$v + \lambda = C_1.$$

(D.14)

Lalu

$$e^{-\lambda}\left\{\frac{r(v' - \lambda')}{2} + 1\right\} - 1 = 0$$

$$e^{-\lambda}\left\{\frac{r(-\lambda' - \lambda')}{2} + 1\right\} - 1 = 0$$

$$e^{-\lambda}\left\{-\frac{r(2\lambda')}{2} + 1\right\} - 1 = 0$$

$$e^{-\lambda}\{1 - r\lambda'\} = 1$$

$$\frac{\partial}{\partial r}\{re^{-\lambda}\} = 1.$$

(D.15)

Kita integralkan

$$\int \frac{\partial}{\partial r}\{re^{-\lambda}\}dr = \int 1\, dr$$
$$re^{-\lambda} = r + C_2$$
$$e^{-\lambda} = 1 + \frac{C_2}{r}.$$

(D.16)

Untuk menentukan solusi konstanta integrasi C_1, kita berikan syarat batas $(r \approx \infty)$, sehingga elemen garis ds^2 di atas haruslah mereduksi ke bentuk Minkowskian. Dengan kata lain

$$g_{00}(r \approx \infty) = e^{v} = 1$$
$$g_{11}(r \approx \infty) = e^{\lambda} = -1$$

(D.17)

yang hanya dipenuhi bila $v(r \approx \infty) = 0$ dan $\lambda(r \approx \infty) = 0$, karena $e^{\infty} = e^{0} = 1$. Sehingga $C_1 = 0$ dan

$$v + \lambda = C_1 = 0$$
$$v = -\lambda.$$

(D.18)

Untuk menentukan C_2, kita berikan apa yang disebut sebagai *limit medan lemah* (yaitu limit Newtonian dari TRU)

$$g_{00} \cong 1 + \frac{2}{c^2}\emptyset$$

(D.19)

di mana *c* adalah kecepatan cahaya dan $\emptyset = -\frac{GM}{r}$ adalah potensial gravitasi. Sementara itu kita tahu bahwa $g_{00} = e^{v}$ dan $v = -\lambda$, sehingga

$$g_{00} \cong 1 + \frac{2}{c^2}\emptyset = e^{v} = 1 + \frac{C_2}{r}$$

(D.20)

atau

$$C_2 = -\frac{2r}{c^2}\frac{GM}{r} = -\frac{2GM}{c^2}.$$

(D.21)

Akhirnya didapat

$$e^{v} = 1 + \frac{C_2}{r} = 1 + \frac{\left(-\frac{2GM}{c^2}\right)}{r} = 1 - \frac{2GM}{c^2 r}$$

(D.22)

dan

$$e^{-\lambda} = 1 + \frac{C_2}{r} = 1 + \frac{\left(-\frac{2GM}{c^2}\right)}{r} = 1 - \frac{2GM}{c^2 r}$$

$$e^{\lambda} = \frac{1}{\left(1 - \frac{2GM}{c^2 r}\right)}.$$

(D.23)

Dengan demikian, *ruang-waktu Schwarzschild* kita dapatkan, yakni

$$ds^2 = \left(1 - \frac{2GM}{c^2 r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2GM}{c^2 r}\right)} - r^2(d\theta^2 + sin^2\theta d\varphi^2).$$

(D.24)

Metrik ini menggambarkan medan gravitasi di luar objek simetri bola yang tidak bergantung pada distribusi materi di dalam objek (Hidayat, 2010; Purwanto, 2009).