MODEL MATEMATIKA PENYAKIT DIABETES MELLITUS TANPA FAKTOR GENETIK DENGAN PERAWATAN

Skripsi

Untuk memenuhi sebagian persyaratan

mencapai derajat Sarjana S-1

Program Studi Matematika

diajukan oleh

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Kepada

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YOGYAKARTA

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Assalamu'alaikum wr. wb.

Setelah membaca, meneliti, memberikan petunjuk dan mengoreksi serta mengadakan perbaikan seperlunya, maka kami selaku pembimbing berpendapat bahwa skripsi Saudara:

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Dengan ini kami mengharap agar skripsi/tugas akhir Saudara tersebut di atas dapat segera dimunaqsyahkan. Atas perhatiannya kami ucapkan terima kasih.

Wassalamu'alaikum wr. wb.

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Menyatakan dengan sesungguhnya bahwa skripsi ini merupakan hasil pekerjaan penulis sendiri dan sepanjang pengetahuan penulis tidak berisi materi yang dipublikasikan atau ditulis orang lain, dan atau telah digunakan sebagai persyaratan penyelesaian Tugas Akhir di Perguruan Tinggi lain, kecuali bagian tertentu yang penulis ambil sebagai bahan acuan. Apabila terbukti pernyataan ini tidak benar, sepenuhnya menjadi tanggung jawab penulis.

Yogyakarta, 9 Juni 2016

Yang menyatakan

METERAI TEMPEL

6000

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Karya sederhana dipersembahkan untuk

Bapak dan Ibu tercinta.

Mas, Mbak, dan keponakanku tersayang

dan

Calon Imamku

Barang siapa bersungguh-sungguh, sesungguhnya kesungguhannya itu adalah untuk dirinya sendiri.

(Al-Ankabut : 6)

Karena sesungguhnya sesudah kesulitan itu ada kemudahan

(Ash-sharh ayat 5)

You only live once, But if you do it right, Once is enough.

(Mae West)

Jangan hanya menjalani hidup, tetapi berkembanglah dengan kehidupan

(Penulis)

PRAKATA

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Penelitian ini membahas tentang penyakit Diabetes Mellitus yang dimodelkan dalam persamaan matematika sehingga diperoleh titik ekuilibrium dimana penyebaran penyakit stabil dengan menggunakan persamaan differensial. Selanjutnya dilakukan simulasi numerik untuk mengetahui gambaran dari model matematika untuk penyakit Diabetes Mellitus tersebut. Harapan penulis semoga penelitian ini dapat menambah wawasan dan pengetahuan bagi pembaca pada umumnya dan bagi peneliti khususnya.

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Wassalamu’alaikum Wr. Wb.

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Penulis

DAFTAR ISI

DAFTAR TABEL

DAFTAR LAMBANG

A	= Laju perekrutan manusia populasi manusia
μ	= Laju kematian alami pada populasi manusia tiap kompertemen
β	= Laju kontak invektif individu yang rentan terhadap individu laten
$\alpha\gamma$	= Laju perpindahan individu laten terhadap individu sakit tanpa perawatan
$(1-\alpha\gamma)$	= Laju perpindahan individu laten terhadap individu sakit dengan perawatan
δ_1	= Laju kematian akibat penyakit tanpa perawatan
δ_2	= Laju kematian akibat penyakit dengan perawatan
$\mathbb{R}$	= Himpunan bilangan riil
$\mathbb{R}^n$	= Himpunan bilangan riil di dimensi n
λ	= Nilai eigen
$\hat{x}$	= Titik Ekuilibrium
$J_f(x)$	= Matriks Jacobian dari f pada titik x
$J_f(\hat{x})x$	= Linearisasi $f(x)$ di sekitar $\hat{x}$
C^n	= Fungsi yang selalu kontinu hingga turunan ke-n
ρ	= radius spektral (nilai eigen terbesar)
$(\mathbb{R}_0^+)^4$	= Daerah non negatif di $\mathbb{R}^4$
K	= *Next generation matrix*
R_0	= Bilangan Reproduksi dasar

INTISARI

MODEL MATEMATIKA PENYAKIT DIABETES MELLITUS TANPA FAKTOR GENETIK DENGAN PERAWATAN

Oleh

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Diabetes Mellitus adalah gangguan metabolisme yang ditandai dengan kenaikan kadar glukosa dalam darah (hiperglikemi), yang menimbulkan berbagai komplikasi kronik yang disebabkan oleh kelainan sekresi insulin. Diabetes Mellitus dikenal sebagai *Mother of Disease* karena merupakan induk dari berbagai penyakit lainnya seperti hipertensi, penyakit jantung, stroke dan kebutaan. Dalam penulisan ini yang akan dikaji adalah model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan, model yang digunakan untuk pendekatan dalam kasus ini berbentuk $SEII_T$. Analisa yang dilakukan meliputi penentuan titik ekuilibrium model, penentuan *basic reproduction number* (R_0) dan analisa kestabilan disekitar titik ekuilibrium . Selanjutnya, simulasi dengan menggunakan MAPLE diberikan berdasarkan nilai - nilai parameter yang terkait dalam model matematika yang menggambarkan kondisi pada setiap kelas subpopulasi.

Kata kunci: $SEII_T$, persamaan diferensial, titik ekuilibrium, stabil asimtotik.

ABSTRACT

DIABETES MELLITUS MATHEMATICS MODELING WITHOUT GENETIC FACTORS USING TREATMENTS

By

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Diabetes mellitus is a metabolism abnormality which has indication of the rising Glucose level in blood (hyperglycemia) that generates various chronicle complications caused by insulin secretion anomaly. Diabetes mellitus is known by Mother of Disease because this disease is the essence of other diseases such as hypertension, cardiac disease, stroke, and blindness. This paper will discuss about Diabetes Mellitus Mathematics Modeling without Genetic Factors Using Treatments, the approaching methods in this case uses $SEII_T$ form. The analysis technique involves the equilibrium point determining model, basic reproduction number (R_0) determining, and the stability around the equilibrium point analysis. After that, the simulation uses MAPLE based on the parameter values which are related in Mathematics modeling that represent the conditions each subpopulation class.

Key word: $SEII_T$, differential equation, equilibrium point, asymptotically stable.

BAB I

PENDAHULUAN

1.1. Latar Belakang Masalah

Penyakit Gula atau Kencing Manis yang dalam istilah medisnya disebut Diabetes Mellitus (DM) adalah gangguan metabolisme yang ditandai dengan kenaikan kadar glukosa dalam darah atau hiperglikemi, yang menimbulkan berbagai komplikasi kronik yang disebabkan oleh kelainan sekresi insulin, kerja insulin atau kedua-duanya. Diabetes Mellitus juga sering disebut sebagai *the great imitator* karena penyakit ini dapat mengenai semua organ tubuh dan menimbulkan berbagai macam keluhan dan gejala yang sangat bervariasi. Diabetes Mellitus dikenal sebagai *Mother of Disease* karena merupakan induk atau ibu dari berbagai penyakit lainnya seperti hipertensi, penyakit jantung, pembuluh darah, stroke, gagal ginjal dan kebutaan.

Penyakit Diabetes Mellitus memiliki dua macam tipe, yaitu: IDDM *(Insulin Dependent Diabetes Mellitus)* dan NIDDM *(Non Insulin Dependent Diabetes Mellitus)*. Beberapa faktor yang dapat mempengaruhi terjangkitnya penyakit ini adalah kurang gerak (malas melakukan kegiatan fisik), makan secara berlebihan, kehamilan, kekurangan hormon insulin dan hormon insulin yang terpacu berlebihan. Penyakit Diabetes Mellitus ini tidak dapat disembuhkan, namun dapat dikendalikan, yaitu dengan menjaga agar kadar gula darah stabil atau mendekati normal, sehingga perlu kesadaran pasien akan pentingnya menjaga pola makan dan hidup sehat.

Sementara itu seiring dengan perkembangan ilmu pengetahuan khususnya

di bidang matematika turut memberikan peranan dalam menganalisis dan memodelkan suatu peristiwa atau permasalahan. Model matematika yang dihasilkan, baik dalam bentuk persamaan, pertidaksamaan, sistem persamaan atau lainnya terdiri atas sekumpulan lambang yang disebut variabel atau besaran yang kemudian didalamnya digunakan operasi matematika seperti tambah, kurang, kali, bagi, dan sebagainya. Dengan prinsip-prinsip matematika tersebut dapat dilihat model yang dihasilkan telah sesuai dengan rumusan sebagaimana formulasi masalah nyata yang dihadapi. Salah satunya adalah model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan.

Model matematika yang digunakan untuk menganalisis penyebaran penyakit diantaranya adalah

- SIR (*Susceptible-Infected-Recovered*)
- $SEIR$ (*Susceptible-Exposed-Infected-Recovered*)
- SEI (*Susceptible-Exposed-ill*)

Dalam karya tulis ini akan dibahas pemodelan matematika penyebaran penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan menggunakan model $SEII_T$ (*Susceptible, Exposed, ILL, ILL with treatment*). Pada model ini populasi dibagi menjadi empat kelompok individu yaitu individu normal (belum terkena diabetes) dimasukkan dalam kelas *Susceptible* (populasi rentan), kelompok individu yang memiliki kebiasaan buruk, penurunan hormon insulin dan peningkatan glukosa darah dimasukkan dalam kelas *Exposed* (populasi laten) dan kelas *ILL* yang mendapatkan perawatan (I_T) dan kelas *ILL* yang tidak mendapatkan perawatan (I). Akan dicari titik ekuilibrium (bebas penyakit dan endemik), selanjutnya akan dicari nilai *Basic Reproduction Number (*R_0*)*, kemudian melakukan analisis kestabilan titik ekuilibrium dan melakukan simulasi dengan menggunakan MAPLE.

1.2. Rumusan Masalah

Berdasarkan uraian di atas, maka dapat dirumuskan beberapa masalah yang akan dibahas. Secara terperinci masalah-masalah yang dimaksud mencakup hal-hal sebagai berikut:

1. Bagaimana mengetahui titik ekuilibrium bebas penyakit dan titik ekuilibrium endemik dari model penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan?
2. Bagaimana mengetahui nilai *Basic Reproduction Number* (R_0) dari model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan?
3. Bagaimana mengetahui analisis kestabilan titik ekuilibrium bebas penyakit dan titik ekuilibrium endemik dari model penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan?
4. Bagaimana simulasi kestabilan titik ekuilibrium model matematika untuk penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan?

1.3. Batasan Masalah

Penulisan skripsi ini dibatasi pada:

1. Penentuan titik ekuilibrium model matematika pada penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan pada jurnal yang ditulis oleh Julia Ulfah, M. Kharis dan Moch Chotim (2013). Model Matematika dan parameter yang digunakan juga berasal dari jurnal tersebut.
2. Angka kelahiran dalam populasi diasumsikan sama dengan angka kematian. Pengaruh migrasi diabaikan sehingga penyebaran penyakit bersifat tertutup

dalam suatu populasi.

1.4. Tujuan

Berdasarkan perumusan masalah, penulisan ini bertujuan untuk:

1. Mengetahui titik ekuilibrium bebas penyakit dan titik ekuilibrium endemik dari model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan.
2. Mengetahui nilai *Basic Reproduction Number*(R_0) dari model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan.
3. Mengetahui analisis kestabilan titik ekuilibrium bebas penyakit dan titik ekuilibrium endemik dari model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan.
4. Mengetahui simulasi kestabilan titik ekuilibrium dari model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan dengan MAPLE.

1.5. Manfaat Penelitian

Mengacu pada tujuan penelitian di atas, maka manfaat penelitian meliputi hal-hal sebagai berikut:

1. Mengetahui titik ekuilibrium bebas penyakit dan titik ekuilibrium endemik dari model penyebaran penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan.
2. Mengetahui nilai *Basic Reproduction Number* (R_0) dari model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan.

3. Mengetahui analis kestabilan titik ekuilibrium bebas penyakit dan titik ekuilibrium endemik dari model penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan.

4. Mengetahui simulasi kestabilan titik ekuilibrium dari model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan dengan MAPLE.

1.6. Tinjauan Pustaka

Penulisan tugas akhir ini mengacu pada literatur-literatur yang tercantum dalam daftar pustaka. Acuan penulisan tugas akhir ini menggunakan beberapa sumber pustaka. Untuk beberapa pengertian dasar aljabar linear tentang nilai eigen, ruang vector, dan transformasi linear mengacu pada Anton (2000). Selanjutnya mengenai matriks Jacobian, titik ekuilibrium, dan linearisasi, serta teorema penting tentang 5 kestabilan system nonlinear mengacu pada Bender (1978), Perko (2001), Oldser (2004),dan Murray (1993). Untuk mencari *Basic Reproduction Ratio* dengan menggunakan rujukan jurnal Driessche dan Watmough (2002). Penulisan tugas akhir Pemodelan Matematika untuk penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan ini merujuk pada jurnal yang ditulis oleh Julia Ulfah, M. Kharis dan Moch Chotim (Model Matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan).

Adapun jurnal dan penelitian yang berkaitan dengan penelitian yang sekarang antara lain: A. Boutayeb (2006) dalam penelitiannya yang berjudul ”A non-linear population Model of Diabetes Mellitus” menjelaskan tentang hubungan antara penderita Diabetes Mellitus tanpa komplikasi dan penderita Diabetes Mellitus dengan komplikasi. Dalam penelitian ini metode numerik yang digunakan adalah metode beda hingga metode Euler.

N. Ardiansah dan M. Kharis (2012) dalam papernya yang berjudul ”Model Matematika untuk Penyakit Diabetes Mellitus Tanpa Faktor Genetik”. Dalam paper ini model matematika yang digunakan adalah model SEI (*Susceptible, Exposed, Ill*). Simulasi numerik yang digunakan adalah dengan menggunakan *software* MATLAB.

Debby Agustine (2013) dalam penelitiannya yang berjudul ” Model Matematika Penyakit Diabetes Dengan Pengaruh Transmisi”, paper ini membahas tentang penyebaran Diabetes Mellitus berdasarkan kandungan darah.

Tabel 1.1 Tinjauan Pustaka

Penulis	Judul	Perbedaan
A.Bautayeb, dkk (2006)	A Non-Linear Population Model of Diabetes Mellitus	Jurnal menjelaskan hubungan antara penderita diabetes yang komplikasi dan tidak.
N.Ardiansah, M. Kharis. (2012)	Model Matematika untuk Penyakit Diabetes Tanpa Faktor Genetik	Jurnal tidak menggunakan perawatan dalam populasi penderita Diabetes Mellitus.
Debby Agustine (UNJ, 2013)	Model Matematika Penyakit Diabetes dengan Pengaruh Transmisi Vertikal	Jurnal menjelaskan model berdasarkan kandungan gula dalam darah.
Nurul Fitriyah (UIN Sunan Kalijaga, 2016)	Model Matematika untuk Penyakit Diabetes Mellitus Tanpa Faktor Genetik dengan Perawatan	Peneliti menggunakan perawatan pada populasi yang sakit.

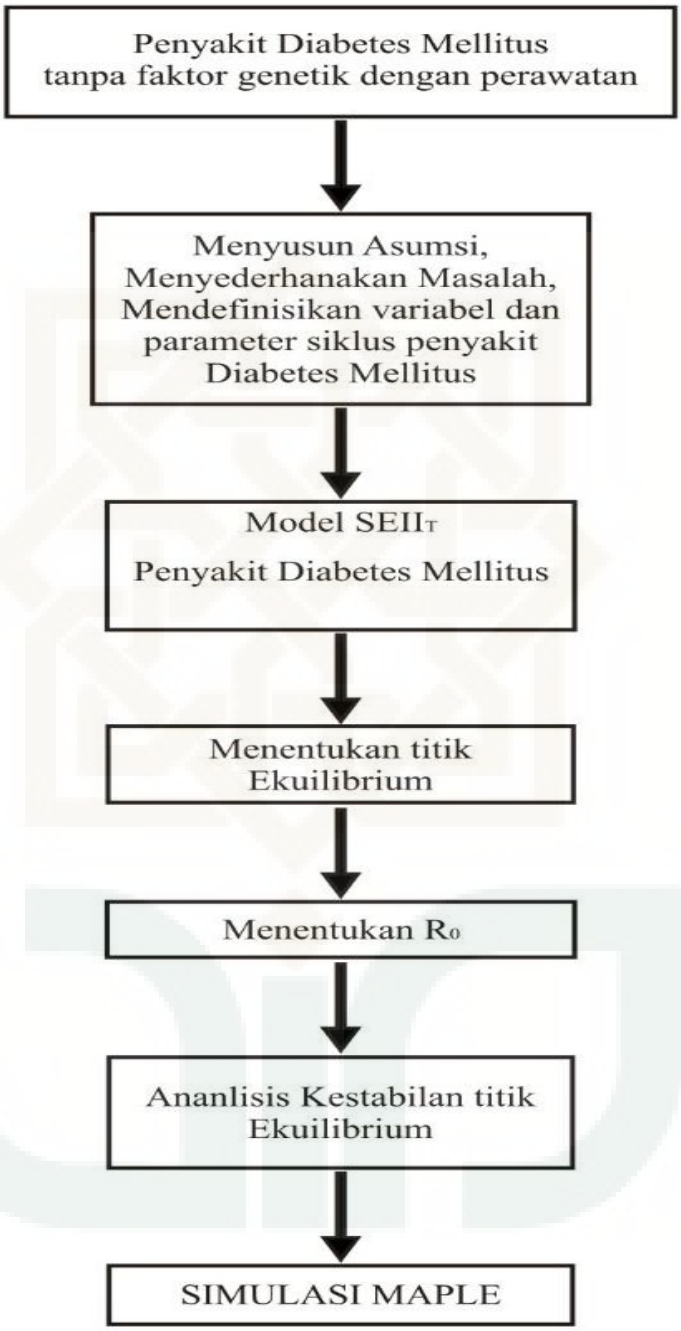
1.7. Metode Penelitian

Metode penelitian yang digunakan dalam penulisan skripsi ini adalah metode studi literatur, yaitu membahas topik masalah secara teoritis dan konseptual. Sumber literatur yang digunakan diperoleh dari jurnal, karya ilmiah dan buku referensi yang menunjang skripsi tentang model matematika penyakit Diabetes

Mellitus tanpa faktor genetik dengan perawatan.

Dari berbagai sumber pustaka yang menjadi bahan kajian, diperoleh suatu pemecahan masalah yang tersebut dalam rumusan masalah, selanjutnya dilakukan langkah - langkah pemecahan masalah sebagai berikut: menjelaskan bagaimana menentukan titik ekuilibrium dan analisis kestabilan model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan, menjelaskan bagaimana menentukan nilai R_0, dan menjelaskan bagaimana simulasi model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan menggunakan program MAPLE.

Hasil dari pembahasan ini dituangkan dalam bentuk simpulan akhir yang menyimpulkan secara umum pemecahan masalah tersebut. Simpulan ini dijadikan sebagai hasil kajian akhir dan merupakan hasil akhir dari proses penulisan ini.



Gambar 1.1 Diagram Alur Metode Penelitian

1.8. Sistematika Penulisan

Sistematika penulisan ini untuk memberikan gambaran menyeluruh dan memudahkan dalam penelitian ini, secara garis besar sistematikanya yaitu:

BAB I : PENDAHULUAN

Pada bab ini dijelaskan latar belakang, rumusan masalah, batasan, tujuan, manfaat, keaslian penelitian, dan sistematika penulisan.

BAB II : LANDASAN TEORI

Pada bab ini dijelaskan teori-teori dan penelitian terdahulu yang digunakan sebagai acuan dan dasar dalam penelitian.

BAB III : MODEL MATEMATIKA DABETES MELLITUS

Pada bab ini dijelaskan mengenai penyakit diabetes mellitus, formulasi model beserta pembahasannya.

BAB IV : SIMULASI NUMERIK

Pada bab ini berisikan simulasi numerik dari pemodelan yang dibahas, sehingga diperoleh gambaran dari hasil penelitian yang dilakukan.

BAB V : PENUTUP

Pada bab ini ditulis kesimpulan akhir dari penelitian dan saran untuk pengembangan penelitian selanjutnya.

BAB V

PENUTUP

Berdasarkan hasil analisis dan simulai model matematika untuk penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan, diperoleh kesimpulan dan saran sebagai berikut :

5.1. Kesimpulan

1. Model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan mempunyai dua titik ekuilibrium yaitu:

   a. Titik ekuilibrium bebas penyakit $P_0 = (N, E, I, I_T) = (\frac{A}{\mu}, 0, 0, 0)$.

   b. Titik ekuilibrium endemik $P_1 = (N^*, E^*, I^*, I_T^*)$ dengan

$$\begin{aligned}
N^* &= \frac{A}{\mu} - \frac{\delta_1\alpha\gamma\mu(\beta A - \mu - \mu^2)}{(\mu+\delta_1)\cdot\beta(1+\mu)} - \frac{\delta_2(1-\alpha\gamma)\mu(\beta A - \mu - \mu^2)}{(\mu+\delta_2)\cdot\beta(1+\mu)} \\
E^* &= \frac{\beta A - \mu(\mu+1)}{\beta(1+\mu)} \\
I^* &= \frac{\alpha\gamma(\beta A - \mu - \mu^2)}{\beta(\mu+\mu^2+\delta_1+\mu\delta_1)} \\
I_T^* &= \frac{(1-\alpha\gamma)(\beta A - \mu - \mu^2)}{\beta(\mu+\mu^2+\delta_2+\delta_2\mu)}
\end{aligned}$$

2. Nilai *Basic Reproduction Number (R_0)* untuk model matematika penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan yaitu $R_0 = \frac{\beta A}{\mu(\mu+1)}$.

3. Titik ekuilibrium bebas penyakit $P_0 = (N, E, I, I_T)$ adalah stabil asimtotik lokal dengan syarat $R_0 < 1$, dan titik ekuilibrium endemik $P_1 = (N^*, E^*, I^*, I_T^*)$ stabil asimtotik lokal dengan syarat $R_0 > 1$.

4. Simulasi MAPLE dengan menggunakan parameter-parameter yang bersesuaian dengan R_0 memperoleh:

   - Simulasi pada titik ekuilibrium bebas penyakit $P_0 = (N,\ E,\ I,\ I_T)$ menunjukkan bahwa N akan stabil pada titik $N(t) = 757.5$, E, I dan I_T akan stabil pada saat titik $E(t) = 0$, $I(t) = 0$ dan $I_T(t) = 0$.
   - Simulasi pada titik ekuilibrium endemik $P_1 = (N^*,\ E^*,\ I^*,\ I_T^*\ )$ menunjukkan bahwa populasi pada setiap subpopulasi akan selalu ada dalam populasi, titik N^* akan menuju kesetimbangan titik N_0^*, dan titik $E^*,\ I^*$ dan I_T^* akan stabil pada saat titik $E^*(t) = 0.5$, $I(t) = 0.8$ dan $I_T(t) = 200$.

5.2. Saran

Pada penelitian model $SEII_T$ untuk penyakit Diabetes Mellitus tanpa faktor genetik dengan perawatan diasumsikan laju kelahiran sama dengan laju kematian. Oleh karena itu, penulis memberikan saran kepada pembaca yang tertarik pada masalah ini untuk mengembangkan model $SEII_T$ dengan memperhatikan laju kelahiran yang tidak sama dengan laju kematian.

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LAMPIRAN A

Print Out *SOFTWARE* MAPLE VERSI 16

1.1. Print out Mapel 16 untuk mencari nilai karakterikstik pada titik ekuilibrium endemik

> restart;

> with(linalg) :

$$> K := matrix\left(4, 4, \left[-\mu, 0, -\delta_1, -\delta_2, \frac{\beta \cdot A}{(1+\mu)} - \mu, \frac{\beta \cdot A}{\mu} - \frac{\delta_1 \cdot \alpha\gamma \cdot \mu \cdot (\beta \cdot A - \mu - \mu^2)}{(\mu+\delta_1)\cdot(1+\mu)} - \frac{\delta_2 \cdot (1-\alpha\gamma)\cdot \mu \cdot (\beta \cdot A - \mu - \mu^2)}{(\mu+\delta_2)\cdot(1+\mu)} - 2 \cdot \frac{(\beta \cdot A - \mu \cdot (\mu+1))}{(1+\mu)} - \frac{\alpha\gamma (\beta \cdot A - \mu - \mu^2)}{(\mu+\delta_1)\cdot(1+\mu)} - \frac{(1-\alpha\gamma)\cdot(\beta \cdot A - \mu - \mu^2)}{(\mu+\delta_2)\cdot(1+\mu)} - \mu - 1, -\frac{\beta \cdot A}{(1+\mu)} + \mu, -\frac{\beta \cdot A}{(1+\mu)} + \mu, 0, \alpha\gamma, -\mu - \delta_1, 0, 0, 1 - \alpha\gamma, 0, -\mu - \delta_2\right]\right);$$

$$K := \left[\left[-\mu, 0, -\delta_1, -\delta_2\right], \left[\frac{\beta A}{1+\mu} - \mu, \frac{\beta A}{\mu} - \frac{\delta_1 \alpha\gamma \mu (\beta A - \mu - \mu^2)}{(\mu+\delta_1)(1+\mu)} - \frac{\delta_2 (1-\alpha\gamma)\mu (\beta A - \mu - \mu^2)}{(\mu+\delta_2)(1+\mu)} - \frac{2(\beta A - \mu(1+\mu))}{1+\mu} - \frac{\alpha\gamma(\beta A - \mu - \mu^2)}{(\mu+\delta_1)(1+\mu)} - \frac{(1-\alpha\gamma)(\beta A - \mu - \mu^2)}{(\mu+\delta_2)(1+\mu)} - \mu - 1, -\frac{\beta A}{1+\mu} + \mu, -\frac{\beta A}{1+\mu} + \mu\right], \left[0, \alpha\gamma, -\mu - \delta_1, 0\right], \left[0, 1 - \alpha\gamma, 0, -\mu - \delta_2\right]\right] \quad (1)$$

> alpha := eigenvals(K);

$$\alpha := -\mu - \delta_2, -\mu - \delta_1, -\frac{1}{2}\frac{1}{\mu(\mu^2 + \mu\delta_2 + \mu^3 + \mu^2\delta_2 + \delta_1\mu + \delta_1\delta_2 + \delta_1\mu^2 + \delta_1\mu\delta_2)}\Big(-\beta A\mu\delta_2 + \beta A\mu^2\delta_2 - \beta A\delta_1\delta_2 + \beta A\delta_1\mu^2 + \mu^3\delta_2\beta A + \delta_1\mu\delta_2 + \beta A\mu^3 - \delta_1\alpha\gamma\mu^4 - \delta_1\alpha\gamma\mu^5$$

$$-\mu^3\delta_2\delta_1 - \mu^4\delta_2\delta_1 + \mu^4\delta_2\alpha\gamma + \mu^5\delta_2\alpha\gamma + \delta_1\mu^2\delta_2 - \alpha\gamma\mu^2\delta_2 - \alpha\gamma\mu^3\delta_2 + \delta_1\alpha\gamma\mu^3 - \mu^4\delta_2 - \mu^5\delta_2 + \mu^3\delta_2 + \mu^2\delta_2 + \beta A\delta_1\mu\delta_2 + \delta_1\alpha\gamma\mu^2 + \delta_1\alpha\gamma\mu^3\beta A + \mu^2\delta_2\beta A\delta_1 - \mu^3\delta_2\alpha\gamma\beta A$$

$$+\alpha\gamma\mu\beta A\delta_2 - \mu\alpha\gamma\beta A\delta_1$$

$$-\Big(-14\mu^4\delta_2^2\alpha\gamma\beta A\delta_1 + 4\alpha\gamma\mu^2\beta A\delta_2^2\delta_1 + 14\delta_1^2\alpha\gamma\mu^4\beta A\delta_2 - 4\mu^2\alpha\gamma\beta A\delta_1^2\delta_2 - 4\mu^3\delta_2\alpha\gamma\beta A\delta_1 - 4\mu^4\delta_2\alpha\gamma\beta A\delta_1 - 2\mu^3\delta_2^2\alpha\gamma\beta A\delta_1 + 2\delta_1^2\alpha\gamma\mu^3\beta A\delta_2 + 8\mu^6\delta_2^2\alpha\gamma\delta_1 - 8\delta_1^2\alpha\gamma\mu^6\delta_2 - 1$$

$$\delta_2^2\alpha\gamma + 2\beta^2A^2\delta_1\mu^2\delta_2^2\alpha\gamma - 2\beta^2A^2\delta_1^2\mu^2\delta_2\alpha\gamma + 2\delta_1^2\alpha\gamma\mu^5\beta^2A^2\delta_2 - 2\delta_1\alpha\gamma^2\mu^6\beta^2A^2\delta_2 + 4\delta_1\alpha\gamma^2\mu^4\beta^2A^2\delta_2 + 2\beta A\delta_1^2\delta_2\alpha\gamma\mu^5 - 2\beta A\delta_1\delta_2^2\mu^5\alpha\gamma - 4\beta^2A^2\delta_1^2\delta_2\alpha\gamma\mu^3 + 4\beta^2A^2\delta_1$$

$$\delta_2^2\mu^3\alpha\gamma - 2\beta^2A^2\delta_1\delta_2^2\alpha\gamma\mu + 2\beta^2A^2\delta_1^2\delta_2\mu\alpha\gamma - 4\mu^8\delta_2\beta A\delta_1\alpha\gamma + 2\mu^6\delta_2\beta^2A^2\delta_1\alpha\gamma - 10\delta_1^2\alpha\gamma\mu^6\delta_2\beta A + 4\delta_1\alpha\gamma^2\mu^7\delta_2\beta A - 8\delta_1\alpha\gamma^2\mu^5\beta A\delta_2 - 4\delta_1^2\alpha\gamma\mu^7\delta_2\beta A$$

$$+4\delta_1\alpha\gamma^2\mu^8\delta_2\beta A-8\delta_1\alpha\gamma^2\mu^6\beta A\delta_2+10\mu^6\delta_2^2\delta_1\alpha\gamma\beta A+4\mu^7\delta_2^2\delta_1\alpha\gamma\beta A+8\beta A\mu^5\delta_2\delta_1\alpha\gamma+8\beta A\mu^6\delta_2\delta_1\alpha\gamma-4\beta^2A^2\mu^4\delta_2\delta_1\alpha\gamma+2\beta^2A^2\mu^2\delta_2\alpha\gamma\delta_1-4\beta A\mu^7\delta_2\delta_1\alpha\gamma$$

$$+12\mu^8+24\delta_1\mu^8+12\delta_1^2\mu^7+\delta_1^2\mu^2\delta_2^2-4\beta A\mu^6+48\mu^7\delta_2\delta_1+16\mu^6\delta_2^2\delta_1+24\mu^6\delta_2\delta_1^2+8\mu^5\delta_2^2\delta_1^2+8\mu^7\delta_2^2\alpha\gamma+24\delta_1\mu^7+12\delta_1^2\mu^6+\mu^4\delta_2^2-4\delta_1\alpha\gamma\mu^6-4\delta_1^2\alpha\gamma\mu^5+8\mu^5\delta_2\delta_1$$

$$+4\mu^4\delta_2\delta_1^2+4\mu^6\delta_2\alpha\gamma-8\delta_1\alpha\gamma\mu^7-8\delta_1^2\alpha\gamma\mu^6+2\mu^3\delta_2^2\delta_1^2+32\mu^6\delta_2\delta_1+6\mu^5\delta_2^2\delta_1+16\mu^5\delta_2\delta_1^2+3\mu^4\delta_2^2\delta_1^2+8\mu^7\delta_2\alpha\gamma+10\mu^6\delta_2^2\alpha\gamma+4\mu^4\delta_2^2\delta_1-2\mu^4\delta_2^2\alpha\gamma+24\mu^8\delta_2+8\mu^7\delta_2^2$$

$$+2\mu^5\delta_2^2+16\mu^7\delta_2+3\mu^6\delta_2^2+8\delta_1\mu^6+4\delta_1^2\mu^5+4\mu^6\delta_2+4\mu^7-16\delta_1\mu^6\beta A-8\delta_1^2\mu^5\beta A-20\mu^5\delta_2\beta A\delta_1+4\mu^4\delta_2^2\beta A\delta_1+6\delta_1\alpha\gamma\mu^6\beta A+6\delta_1^2\alpha\gamma\mu^5\beta A-10\mu^4\delta_2\beta A\delta_1^2$$

$$+2\mu^3\delta_2^2\beta A\delta_1^2-6\mu^6\delta_2\alpha\gamma\beta A-14\mu^5\delta_2^2\alpha\gamma\beta A+4\alpha\gamma\mu^3\beta A\delta_2^2-8\delta_1\mu^5\beta A-4\delta_1^2\mu^4\beta A-4\mu^4\delta_2\beta A\delta_1-2\beta A\delta_1^2\mu\delta_2^2+6\delta_1\alpha\gamma\mu^5\beta A+6\delta_1^2\alpha\gamma\mu^4\beta A-2\mu^3\delta_2\beta A\delta_1^2$$

$$-6\mu^5\delta_2\alpha\gamma\beta A-2\mu^4\delta_2^2\alpha\gamma\beta A-4\mu^2\delta_2^2\beta A\delta_1+\alpha\gamma^2\mu^4\delta_2^2+2\alpha\gamma^2\mu^5\delta_2^2+2\delta_1^2\alpha\gamma^2\mu^5+\delta_1^2\alpha\gamma^2\mu^4+8\mu^9\delta_1\alpha\gamma+32\mu^8\delta_2\delta_1-8\mu^9\delta_2\alpha\gamma+4\mu^{10}\delta_1\alpha\gamma+8\mu^9\delta_2\delta_1-4\mu^{10}\delta_2\alpha\gamma+8$$

$$\delta_1^2\mu^8\alpha\gamma+16\delta_1^2\mu^7\delta_2+4\delta_1^2\mu^9\alpha\gamma+4\delta_1^2\mu^8\delta_2+\mu^6\delta_2^2\alpha\gamma^2\beta^2A^2-2\mu^4\delta_2^2\alpha\gamma^2\beta^2A^2+\alpha\gamma^2\mu^2\beta^2A^2\delta_2^2+\mu^2\alpha\gamma^2\beta^2A^2\delta_1^2-2\alpha\gamma^2\mu^3\delta_2^2\beta A-2\alpha\gamma^2\mu^4\delta_2^2\beta A-2\delta_1^2\alpha\gamma^2\mu^4\beta A+2\beta^2A^2\delta_1^2\mu^3$$

$$\delta_2^2-2\delta_1^2\alpha\gamma^2\mu^3\beta A+\delta_1^2\alpha\gamma^2\mu^6\beta^2A^2-2\delta_1^2\alpha\gamma^2\mu^4\beta^2A^2+\mu^4\delta_2^2\beta^2A^2\delta_1^2-4\mu^7\delta_2^2\beta A\delta_1+4\mu^8\delta_2^2\beta A\alpha\gamma+2\mu^5\delta_2^2\beta^2A^2\delta_1-2\mu^6\delta_2^2\beta^2A^2\alpha\gamma-6\beta A\mu^7\delta_1\alpha\gamma-6\beta A\mu^8\delta_1\alpha\gamma$$

$$+6\beta A\mu^7\delta_2\alpha\gamma+6\beta A\mu^8\delta_2\alpha\gamma+2\beta^2A^2\mu^6\delta_1\alpha\gamma-2\beta^2A^2\mu^5\delta_2\alpha\gamma+2\beta^2A^2\mu^4\alpha\gamma\delta_2-2\beta^2A^2\mu^4\alpha\gamma\delta_1-2\delta_1^2\alpha\gamma^2\mu^7\beta A+4\delta_1^2\alpha\gamma^2\mu^5\beta A-2\delta_1^2\alpha\gamma^2\mu^8\beta A+4\delta_1^2\alpha\gamma^2\mu^6\beta A-8\mu^5$$

$$\delta_2^2\delta_1^2\beta A-2\mu^6\delta_2^2\delta_1^2\beta A-2\mu^7\delta_2^2\alpha\gamma^2\beta A+4\mu^5\delta_2^2\alpha\gamma^2\beta A-2\mu^8\delta_2^2\alpha\gamma^2\beta A+4\mu^6\delta_2^2\alpha\gamma^2\beta A+2\beta^2A^2\mu\delta_2^2\delta_1-4\beta^2A^2\mu^3\delta_2\delta_1-12\beta A\mu^5\delta_2^2\delta_1-2\beta A\mu^6\delta_2^2\alpha\gamma-4\beta^2A^2\mu^2\delta_2^2\delta_1$$

$$-2\beta^2A^2\mu^3\delta_2^2\delta_1+4\beta^2A^2\mu^4\delta_2^2\alpha\gamma-2\beta^2A^2\mu^2\delta_2^2\alpha\gamma+4\beta^2A^2\mu^4\delta_2\delta_1-16\beta A\mu^6\delta_2^2\delta_1+10\beta A\mu^7\delta_2^2\alpha\gamma+4\beta^2A^2\mu^4\delta_2^2\delta_1-2\beta^2A^2\mu^5\delta_2^2\alpha\gamma+2\beta^2A^2\mu^3\delta_2^2\alpha\gamma-2\beta^2A^2\delta_1^2\delta_2\mu^2$$

$$-6\beta A\delta_1^2\delta_2^2\mu^4-2\beta^2A^2\delta_1^2\delta_2^2\mu-\beta^2A^2\delta_1^2\delta_2^2\mu^2+4\beta^2A^2\delta_1\mu^5\delta_2-6\beta A\delta_1^2\mu^6\alpha\gamma-6\beta A\delta_1^2\mu^7\alpha\gamma-14\beta A\delta_1^2\mu^5\delta_2-6\beta A\delta_1^2\mu^6\delta_2-28\beta A\delta_1\mu^6\delta_2-12\beta A\delta_1\mu^7\delta_2+2\beta^2A^2$$

$$\delta_1^2\mu^3\delta_2+2\beta^2A^2\delta_1^2\mu^5\alpha\gamma+2\beta^2A^2\delta_1^2\mu^4\delta_2-2\beta^2A^2\delta_1^2\mu^3\alpha\gamma+4\delta_1^2\mu^8+11\mu^8\delta_2^2+6\mu^9\delta_2^2+\mu^{10}\delta_2^2+16\mu^9\delta_2+4\mu^{10}\delta_2+8\mu^9\delta_1-8\mu^7\beta A\delta_1-4\delta_1^2\mu^6\beta A+6\delta_1^2\alpha\gamma\mu^7\delta_2+8$$

$$\delta_1^2\alpha\gamma\mu^8\delta_2+2\delta_1\alpha\gamma^2\mu^8\delta_2-4\delta_1\alpha\gamma^2\mu^9\delta_2+2\delta_1\alpha\gamma^2\mu^6\delta_2+8\delta_1\alpha\gamma^2\mu^7\delta_2-2\delta_1\alpha\gamma\mu^8\delta_2+4\delta_1\alpha\gamma\mu^9\delta_2-8\delta_1\alpha\gamma\mu^7\delta_2+2\delta_1^2\alpha\gamma\mu^9\delta_2-2\delta_1\alpha\gamma^2\mu^{10}\delta_2+2\delta_1\alpha\gamma\mu^{10}\delta_2-6\mu^7\delta_2^2\delta_1\alpha\gamma$$

$$-8\mu^8\delta_2^2\delta_1\alpha\gamma-2\mu^9\delta_2^2\delta_1\alpha\gamma-4\alpha\gamma^2\mu^5\delta_2\delta_1-2\alpha\gamma^2\mu^4\delta_2\delta_1+\beta^2A^2\mu^2\delta_2^2-2\beta^2A^2\mu^3\delta_2^2-\beta^2A^2\mu^4\delta_2^2-2\beta^2A^2\mu^4\delta_2-6\beta A\mu^6\delta_2^2+2\beta^2A^2\mu^5\delta_2^2+2\beta^2A^2\mu^5\delta_2-8\beta A\mu^7\delta_2^2$$

$$+\beta^2A^2\delta_1^2\delta_2^2+\beta^2A^2\delta_1^2\mu^4+2\beta^2A^2\delta_1\mu^5+\mu^6\delta_2^2\beta^2A^2+2\mu^6\delta_2\beta^2A^2-2\mu^8\delta_2^2\beta A-14\beta A\mu^7\delta_2-6\beta A\mu^8\delta_2-\delta_1^2\alpha\gamma^2\mu^8+2\delta_1^2\alpha\gamma^2\mu^9-4\delta_1^2\alpha\gamma^2\mu^7-\delta_1^2\alpha\gamma^2\mu^6+\delta_1^2\alpha\gamma^2\mu^{10}+11\mu^6$$

$$\delta_2^2\delta_1^2+6\mu^7\delta_2^2\delta_1^2+22\mu^7\delta_2^2\delta_1+12\mu^8\delta_2^2\delta_1+\mu^8\delta_2^2\delta_1^2+2\mu^9\delta_2^2\delta_1-\mu^8\delta_2^2\alpha\gamma^2+2\mu^9\delta_2^2\alpha\gamma^2-\mu^6\delta_2^2\alpha\gamma^2-4\mu^7\delta_2^2\alpha\gamma^2-6\mu^8\delta_2^2\alpha\gamma-8\mu^9\delta_2^2\alpha\gamma+\mu^{10}\delta_2^2\alpha\gamma^2-2\mu^{10}\delta_2^2\alpha\gamma+4\mu^{10}+12\mu^9$$

$$+A^2\mu^5\beta^2-8\beta A\mu^7-4\beta A\mu^8\Big)^{1/2}\Big),-\frac{1}{2}\frac{1}{\mu\left(\mu^2+\mu\delta_2+\mu^3+\mu^2\delta_2+\delta_1\mu+\delta_1\delta_2+\delta_1\mu^2+\delta_1\mu\delta_2\right)}\Big(-\beta A\mu\delta_2+\beta A\mu^2\delta_2-\beta A\delta_1\delta_2+\beta A\delta_1\mu^2+\mu^3\delta_2\beta A+\delta_1\mu\delta_2$$

$$+\beta A\mu^3-\delta_1\alpha\gamma\mu^4-\delta_1\alpha\gamma\mu^5-\mu^3\delta_2\delta_1-\mu^4\delta_2\delta_1+\mu^4\delta_2\alpha\gamma+\mu^5\delta_2\alpha\gamma+\delta_1\mu^2\delta_2-\alpha\gamma\mu^2\delta_2-\alpha\gamma\mu^3\delta_2+\delta_1\alpha\gamma\mu^3-\mu^4\delta_2-\mu^5\delta_2+\mu^3\delta_2+\mu^2\delta_2+\beta A\delta_1\mu\delta_2+\delta_1\alpha\gamma\mu^2+\delta_1\alpha\gamma\mu^3\beta A$$

$$+\mu^2\delta_2\beta A\delta_1-\mu^3\delta_2\alpha\gamma\beta A+\alpha\gamma\mu\beta A\delta_2-\mu\alpha\gamma\beta A\delta_1$$

$$+\Big(-14\mu^4\delta_2^2\alpha\gamma\beta A\delta_1+4\alpha\gamma\mu^2\beta A\delta_2^2\delta_1+14\delta_1^2\alpha\gamma\mu^4\beta A\delta_2-4\mu^2\alpha\gamma\beta A\delta_1^2\delta_2-4\mu^3\delta_2\alpha\gamma\beta A\delta_1-4\mu^4\delta_2\alpha\gamma\beta A\delta_1-2\mu^3\delta_2^2\alpha\gamma\beta A\delta_1+2\delta_1^2\alpha\gamma\mu^3\beta A\delta_2+8\mu^6\delta_2^2\alpha\gamma\delta_1-8\delta_1^2\alpha\gamma\mu^5\delta_2-$$

$$\delta_2^2\alpha\gamma+2\beta^2A^2\delta_1\mu^2\delta_2^2\alpha\gamma-2\beta^2A^2\delta_1^2\mu^2\delta_2\alpha\gamma+2\delta_1^2\alpha\gamma\mu^5\beta^2A^2\delta_2-2\delta_1\alpha\gamma^2\mu^6\beta^2A^2\delta_2+4\delta_1\alpha\gamma^2\mu^4\beta^2A^2\delta_2+2\beta A\delta_1^2\delta_2\alpha\gamma\mu^5-2\beta A\delta_1\delta_2^2\mu^5\alpha\gamma-4\beta^2A^2\delta_1^2\delta_2\alpha\gamma\mu^3+4\beta^2A^2\delta_1$$

$$\delta_2^2\mu^3\alpha\gamma-2\beta^2A^2\delta_1\delta_2^2\alpha\gamma\mu+2\beta^2A^2\delta_1^2\delta_2\mu\alpha\gamma-4\mu^8\delta_2\beta A\delta_1\alpha\gamma+2\mu^6\delta_2\beta^2A^2\delta_1\alpha\gamma-10\delta_1^2\alpha\gamma\mu^6\delta_2\beta A+4\delta_1\alpha\gamma^2\mu^7\delta_2\beta A-8\delta_1\alpha\gamma^2\mu^5\beta A\delta_2-4\delta_1^2\alpha\gamma\mu^7\delta_2\beta A$$

$$+4\delta_1\alpha\gamma^2\mu^8\delta_2\beta A-8\delta_1\alpha\gamma^2\mu^6\beta A\delta_2+10\mu^6\delta_2^2\delta_1\alpha\gamma\beta A+4\mu^7\delta_2^2\delta_1\alpha\gamma\beta A+8\beta A\mu^5\delta_2\delta_1\alpha\gamma+8\beta A\mu^6\delta_2\delta_1\alpha\gamma-4\beta^2A^2\mu^4\delta_2\delta_1\alpha\gamma+2\beta^2A^2\mu^2\delta_2\alpha\gamma\delta_1-4\beta A\mu^7\delta_2\delta_1\alpha\gamma$$

$$+12\mu^8+24\delta_1\mu^8+12\delta_1^2\mu^7+\delta_1^2\mu^2\delta_2^2-4\beta A\mu^6+48\mu^7\delta_2\delta_1+16\mu^6\delta_2^2\delta_1+24\mu^6\delta_2\delta_1^2+8\mu^5\delta_2^2\delta_1^2+8\mu^7\delta_2^2\alpha\gamma+24\delta_1\mu^7+12\delta_1^2\mu^6+\mu^4\delta_2^2-4\delta_1\alpha\gamma\mu^6-4\delta_1^2\alpha\gamma\mu^5+8\mu^5\delta_2\delta_1$$

$$+4\mu^4\delta_2\delta_1^2+4\mu^6\delta_2\alpha\gamma-8\delta_1\alpha\gamma\mu^7-8\delta_1^2\alpha\gamma\mu^6+2\mu^3\delta_2^2\delta_1^2+32\mu^6\delta_2\delta_1+6\mu^5\delta_2^2\delta_1+16\mu^5\delta_2\delta_1^2+3\mu^4\delta_2^2\delta_1^2+8\mu^7\delta_2\alpha\gamma+10\mu^6\delta_2^2\alpha\gamma+4\mu^4\delta_2^2\delta_1-2\mu^4\delta_2^2\alpha\gamma+24\mu^8\delta_2+8\mu^7\delta_2^2$$

$$+2\mu^5\delta_2^2+16\mu^7\delta_2+3\mu^6\delta_2^2+8\delta_1\mu^6+4\delta_1^2\mu^5+4\mu^6\delta_2+4\mu^7-16\delta_1\mu^6\beta A-8\delta_1^2\mu^5\beta A-20\mu^5\delta_2\beta A\delta_1+4\mu^4\delta_2^2\beta A\delta_1+6\delta_1\alpha\gamma\mu^6\beta A+6\delta_1^2\alpha\gamma\mu^5\beta A-10\mu^4\delta_2\beta A\delta_1^2$$

$$+2\mu^3\delta_2^2\beta A\delta_1^2-6\mu^6\delta_2\alpha\gamma\beta A-14\mu^5\delta_2^2\alpha\gamma\beta A+4\alpha\gamma\mu^3\beta A\delta_2^2-8\delta_1\mu^5\beta A-4\delta_1^2\mu^4\beta A-4\mu^4\delta_2\beta A\delta_1-2\beta A\delta_1^2\mu\delta_2^2+6\delta_1\alpha\gamma\mu^5\beta A+6\delta_1^2\alpha\gamma\mu^4\beta A-2\mu^3\delta_2\beta A\delta_1^2$$

$$-6\mu^5\delta_2\alpha\gamma\beta A-2\mu^4\delta_2^2\alpha\gamma\beta A-4\mu^2\delta_2^2\beta A\delta_1+\alpha\gamma^2\mu^4\delta_2^2+2\alpha\gamma^2\mu^5\delta_2^2+2\delta_1^2\alpha\gamma^2\mu^5+\delta_1^2\alpha\gamma^2\mu^4+8\mu^9\delta_1\alpha\gamma+32\mu^8\delta_2\delta_1-8\mu^9\delta_2\alpha\gamma+4\mu^{10}\delta_1\alpha\gamma+8\mu^9\delta_2\delta_1-4\mu^{10}\delta_2\alpha\gamma+8$$

$$\delta_1^2\mu^8\alpha\gamma+16\delta_1^2\mu^7\delta_2+4\delta_1^2\mu^9\alpha\gamma+4\delta_1^2\mu^8\delta_2+\mu^6\delta_2^2\alpha\gamma^2\beta^2A^2-2\mu^4\delta_2^2\alpha\gamma^2\beta^2A^2+\alpha\gamma^2\mu^2\beta^2A^2\delta_2^2+\mu^2\alpha\gamma^2\beta^2A^2\delta_1^2-2\alpha\gamma^2\mu^3\delta_2^2\beta A-2\alpha\gamma^2\mu^4\delta_2^2\beta A-2\delta_1^2\alpha\gamma^2\mu^4\beta A+2\beta^2A^2\delta_1^2\mu^3$$

$$\delta_2^2-2\delta_1^2\alpha\gamma^2\mu^3\beta A+\delta_1^2\alpha\gamma^2\mu^6\beta^2A^2-2\delta_1^2\alpha\gamma^2\mu^4\beta^2A^2+\mu^4\delta_2^2\beta^2A^2\delta_1^2-4\mu^7\delta_2^2\beta A\delta_1+4\mu^8\delta_2^2\beta A\alpha\gamma+2\mu^5\delta_2^2\beta^2A^2\delta_1-2\mu^6\delta_2^2\beta^2A^2\alpha\gamma-6\beta A\mu^7\delta_1\alpha\gamma-6\beta A\mu^8\delta_1\alpha\gamma$$

$$+6\beta A\mu^7\delta_2\alpha\gamma+6\beta A\mu^8\delta_2\alpha\gamma+2\beta^2A^2\mu^6\delta_1\alpha\gamma-2\beta^2A^2\mu^5\delta_2\alpha\gamma+2\beta^2A^2\mu^4\alpha\gamma\delta_2-2\beta^2A^2\mu^4\alpha\gamma\delta_1-2\delta_1^2\alpha\gamma^2\mu^7\beta A+4\delta_1^2\alpha\gamma^2\mu^5\beta A-2\delta_1^2\alpha\gamma^2\mu^8\beta A+4\delta_1^2\alpha\gamma^2\mu^6\beta A-8\mu^5$$

$$\delta_2^2\delta_1^2\beta A-2\mu^6\delta_2^2\delta_1^2\beta A-2\mu^7\delta_2^2\alpha\gamma^2\beta A+4\mu^5\delta_2^2\alpha\gamma^2\beta A-2\mu^8\delta_2^2\alpha\gamma^2\beta A+4\mu^6\delta_2^2\alpha\gamma^2\beta A+2\beta^2A^2\mu\delta_2^2\delta_1-4\beta^2A^2\mu^3\delta_2\delta_1-12\beta A\mu^5\delta_2^2\delta_1-2\beta A\mu^6\delta_2^2\alpha\gamma-4\beta^2A^2\mu^2\delta_2^2\delta_1$$

$$-2\beta^2A^2\mu^3\delta_2^2\delta_1+4\beta^2A^2\mu^4\delta_2^2\alpha\gamma-2\beta^2A^2\mu^2\delta_2^2\alpha\gamma+4\beta^2A^2\mu^4\delta_2\delta_1-16\beta A\mu^6\delta_2^2\delta_1+10\beta A\mu^7\delta_2^2\alpha\gamma+4\beta^2A^2\mu^4\delta_2^2\delta_1-2\beta^2A^2\mu^5\delta_2^2\alpha\gamma+2\beta^2A^2\mu^3\delta_2^2\alpha\gamma-2\beta^2A^2\delta_1^2\delta_2\mu^2$$

$$-6\beta A\delta_1^2\delta_2^2\mu^4-2\beta^2A^2\delta_1^2\delta_2^2\mu-\beta^2A^2\delta_1^2\delta_2^2\mu^2+4\beta^2A^2\delta_1\mu^5\delta_2-6\beta A\delta_1^2\mu^6\alpha\gamma-6\beta A\delta_1^2\mu^7\alpha\gamma-14\beta A\delta_1^2\mu^5\delta_2-6\beta A\delta_1^2\mu^6\delta_2-28\beta A\delta_1\mu^6\delta_2-12\beta A\delta_1\mu^7\delta_2+2\beta^2A^2$$

$$\alpha_1^2\mu^3\alpha_2+2\beta^2A^2\alpha_1^2\mu^5\alpha\gamma+2\beta^2A^2\alpha_1^2\mu^4\alpha_2-2\beta^2A^2\alpha_1^2\mu^3\alpha\gamma+4\alpha_1^2\mu^8+11\mu^8\alpha_2^2+6\mu^9\alpha_2^2+\mu^{10}\alpha_2^2+16\mu^9\alpha_2+4\mu^{10}\alpha_2+8\mu^9\alpha_1-8\mu^7\beta A\alpha_1-4\alpha_1^2\mu^6\beta A+6\alpha_1^2\alpha\gamma\mu^7\alpha_2+8$$

$$\alpha_1^2\alpha\gamma\mu^8\alpha_2+2\alpha_1\alpha\gamma^2\mu^8\alpha_2-4\alpha_1\alpha\gamma^2\mu^9\alpha_2+2\alpha_1\alpha\gamma^2\mu^6\alpha_2+8\alpha_1\alpha\gamma^2\mu^7\alpha_2-2\alpha_1\alpha\gamma\mu^8\alpha_2+4\alpha_1\alpha\gamma\mu^9\alpha_2-8\alpha_1\alpha\gamma\mu^7\alpha_2+2\alpha_1^2\alpha\gamma\mu^9\alpha_2-2\alpha_1\alpha\gamma^2\mu^{10}\alpha_2+2\alpha_1\alpha\gamma\mu^{10}\alpha_2-6\mu^7\alpha_1^2\alpha_1\alpha\gamma$$

$$-8\mu^8\alpha_1^2\alpha_1\alpha\gamma-2\mu^9\alpha_1^2\alpha_1\alpha\gamma-4\alpha\gamma^2\mu^5\alpha_2\alpha_1-2\alpha\gamma^2\mu^4\alpha_2\alpha_1+\beta^2A^2\mu^2\alpha_1^2-2\beta^2A^2\mu^3\alpha_1^2-\beta^2A^2\mu^4\alpha_1^2-2\beta^2A^2\mu^4\alpha_2-6\beta A\mu^6\alpha_1^2+2\beta^2A^2\mu^5\alpha_1^2+2\beta^2A^2\mu^5\alpha_2-8\beta A\mu^7\alpha_1^2$$

$$+\beta^2A^2\alpha_1^2\alpha_2^2+\beta^2A^2\alpha_1^2\mu^4+2\beta^2A^2\alpha_1\mu^5+\mu^6\alpha_2^2\beta^2A^2+2\mu^6\alpha_2\beta^2A^2-2\mu^8\alpha_2^2\beta A-14\beta A\mu^7\alpha_2-6\beta A\mu^8\alpha_2-\alpha_1^2\alpha\gamma^2\mu^8+2\alpha_1^2\alpha\gamma^2\mu^9-4\alpha_1^2\alpha\gamma^2\mu^7-\alpha_1^2\alpha\gamma^2\mu^6+\alpha_1^2\alpha\gamma^2\mu^{10}+11\mu^6$$

$$\alpha_2^2\alpha_1^2+6\mu^7\alpha_2^2\alpha_1^2+22\mu^7\alpha_2^2\alpha_1+12\mu^8\alpha_2^2\alpha_1+\mu^8\alpha_2^2\alpha_1^2+2\mu^9\alpha_2^2\alpha_1-\mu^8\alpha_2^2\alpha\gamma^2+2\mu^9\alpha_2^2\alpha\gamma^2-\mu^6\alpha_2^2\alpha\gamma^2-4\mu^7\alpha_2^2\alpha\gamma^2-6\mu^8\alpha_2^2\alpha\gamma-8\mu^9\alpha_2^2\alpha\gamma+\mu^{10}\alpha_2^2\alpha\gamma^2-2\mu^{10}\alpha_2^2\alpha\gamma+4\mu^{10}+12\mu^9$$

$$\left.+A^2\mu^6\beta^2-8\beta A\mu^7-4\beta A\mu^8\right)^{1/2}\Big)$$

1.2. Print out Mapel 16 simulasi model untuk $R_0 < 1$

Pada simulasi ini variabel pada sistem (3.10) yaitu: $N(t), E(t), I(t)$ dan $I_T(t)$ dituliskan berturut-turut menjadi $W(t), X(t), Y(t)$ dan $Z(t)$.

1.2.1. Lampiran 1

Simulasi Fase $N(t)$ saat $R_0 < 1$

```
> restart :
> with(DEtools) :
> with(plots) :
> with(linalg) :
> with(student) :
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγ·X − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 750, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 750, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 755, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 755, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 760, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 760, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 765, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 765, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

sol1 := **proc**(x_rkf45) ... **end proc** (13)

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

sol2 := **proc**(x_rkf45) ... **end proc** (14)

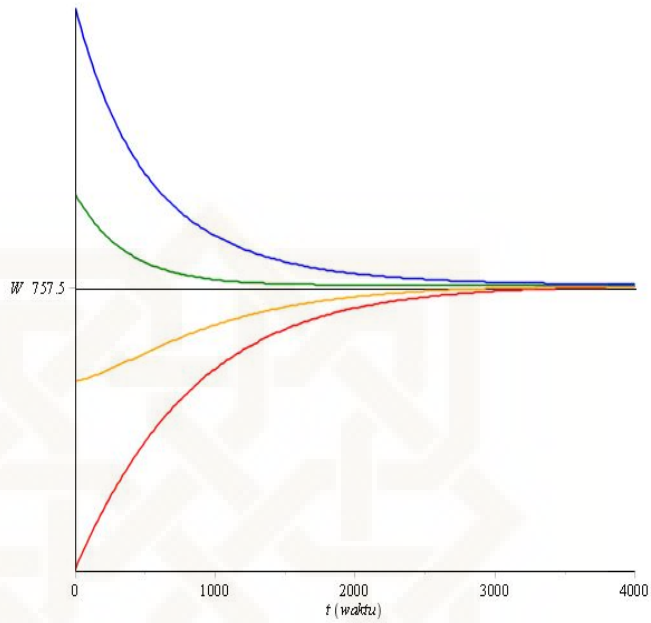
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

sol3 := **proc**(x_rkf45) ... **end proc** (15)

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

sol4 := **proc**(x_rkf45) ... **end proc** (16)

```
> p1 := odeplot(sol1, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```



1.2.2. Lampiran 2

Simulasi Fase $E(t)$ saat $R_0 < 1$

```
> restart :
> with(DEtools) :
> with(plots) :
> with(linalg) :
> with(student) :
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγX − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 750, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 750, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 755, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 755, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 760, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 760, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 765, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 765, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

$sol1 :=$ **proc**(x_rkf45) ... **end proc** (13)

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

$sol2 :=$ **proc**(x_rkf45) ... **end proc** (14)

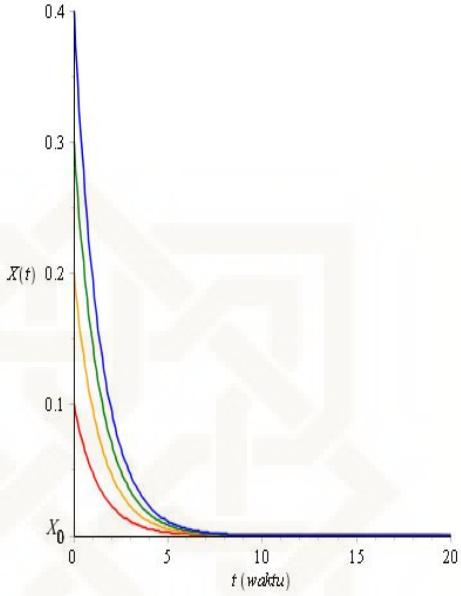
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

$sol3 :=$ **proc**(x_rkf45) ... **end proc** (15)

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

$sol4 :=$ **proc**(x_rkf45) ... **end proc** (16)

```
> p1 := odeplot(sol1, [t, X(t)], 0 ..20, legend = ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, X(t)], 0 ..20, legend = ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, X(t)], 0 ..20, legend = ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, X(t)], 0 ..20, legend = ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```



1.2.3. Lampiran 3

Simulasi Fase $I(t)$ saat $R_0 < 1$

```
> restart :
> with(DEtools) :
> with(plots) :
> with(linalg) :
> with(student) :
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγX − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 750, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 750, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 755, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 755, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 760, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 760, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 765, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 765, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

$sol1 :=$ **proc**(x_rkf45) ... **end proc** (13)

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

$sol2 :=$ **proc**(x_rkf45) ... **end proc** (14)

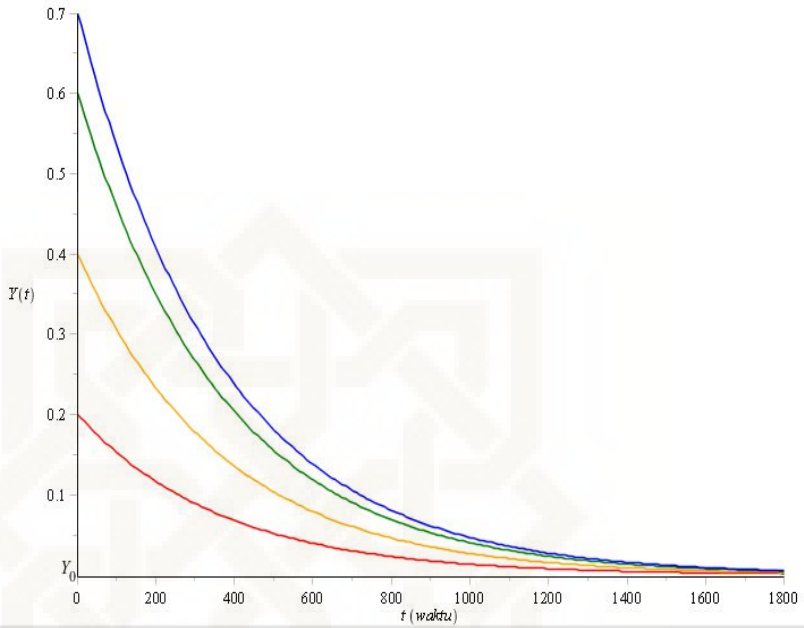
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

$sol3 :=$ **proc**(x_rkf45) ... **end proc** (15)

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

$sol4 :=$ **proc**(x_rkf45) ... **end proc** (16)

```
> p1 := odeplot(sol1, [t, Y(t)], 0 ..1800, legend = ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, Y(t)], 0 ..1800, legend = ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, Y(t)], 0 ..1800, legend = ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, Y(t)], 0 ..1800, legend = ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```



1.2.4. Lampiran 4

Simulasi Fase $IT(t)$ saat $R_0 < 1$

```
> restart :
> with(DEtools) :
> with(plots) :
> with(linalg) :
> with(student) :
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγX − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 750, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 750, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 755, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 755, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 760, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 760, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 1, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 765, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 1, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 765, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

$sol1 :=$ **proc**(x_rkf45) ... **end proc** (13)

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

$sol2 :=$ **proc**(x_rkf45) ... **end proc** (14)

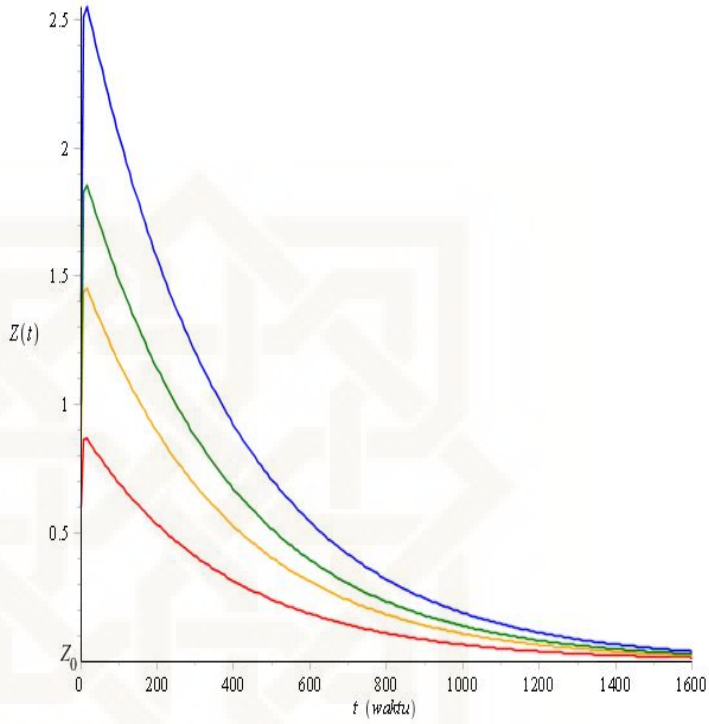
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

$sol3 :=$ **proc**(x_rkf45) ... **end proc** (15)

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

$sol4 :=$ **proc**(x_rkf45) ... **end proc** (16)

```
> p1 := odeplot(sol1, [t, Z(t)], 0 ..1600, legend = ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, Z(t)], 0 ..1600, legend = ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, Z(t)], 0 ..1600, legend = ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, Z(t)], 0 ..1600, legend = ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```




```
>
```

1.3. Print out Mapel 16 simulasi model untuk $R_0 > 1$

Pada simulasi ini variabel pada sistem (3.10) yaitu: $N(t), E(t), I(t)$ dan $I_T(t)$ dituliskan berturut-turut menjadi $W(t), X(t), Y(t)$ dan $Z(t)$.

1.3.1. Lampiran 1

Simulasi Fase $N(t)$ saat $R_0 > 1$

```
> restart:
> with(DEtools):
> with(plots):
> with(linalg):
> with(student):
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγ·X − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1280, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1280, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1290, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1290, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1300, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1300, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1340, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1340, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

$sol1 :=$ **proc**(x_rkf45) ... **end proc** (13)

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

$sol2 :=$ **proc**(x_rkf45) ... **end proc** (14)

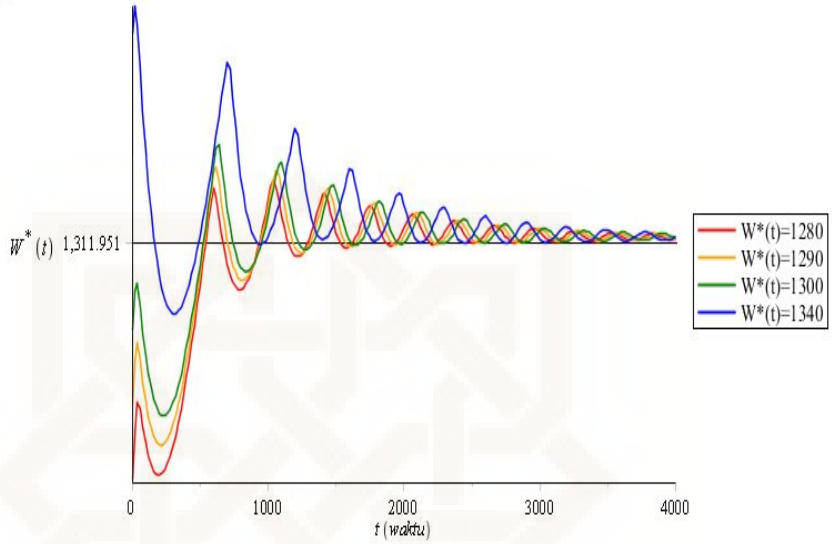
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

$sol3 :=$ **proc**(x_rkf45) ... **end proc** (15)

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

$sol4 :=$ **proc**(x_rkf45) ... **end proc** (16)

```
> p1 := odeplot(sol1, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, W(t)], 0 ..4000, legend = ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```




```
>
```

1.3.2. Lampiran 2

Simulasi Fase $E(t)$ saat $R_0 > 1$

```
> restart :
> with(DEtools) :
> with(plots) :
> with(linalg) :
> with(student) :
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγ·X − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1280, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1280, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1290, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1290, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1300, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1300, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1340, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1340, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

$$sol1 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (13)$$

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

$$sol2 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (14)$$

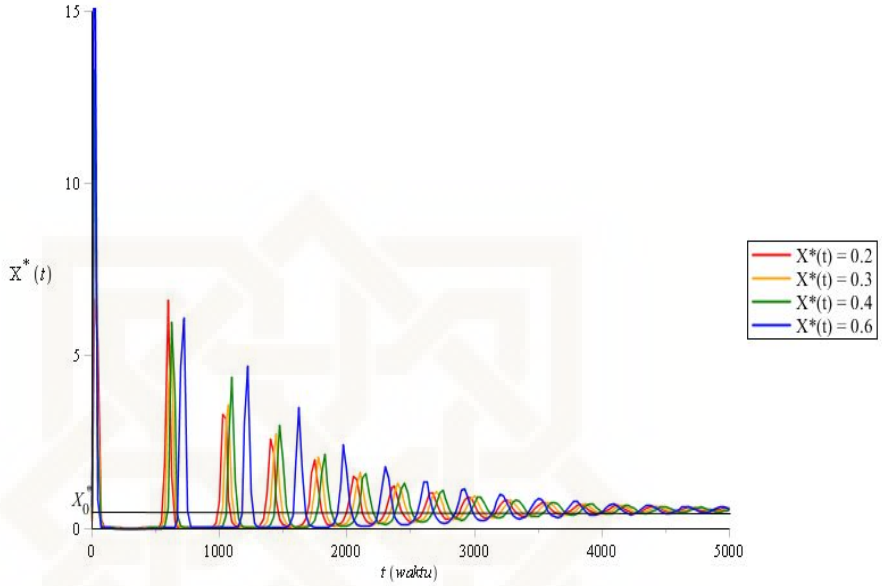
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

$$sol3 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (15)$$

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

$$sol4 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (16)$$

```
> p1 := odeplot(sol1, [t, X(t)], 0 ..5000, legend = ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, X(t)], 0 ..5000, legend = ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, X(t)], 0 ..5000, legend = ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, X(t)], 0 ..5000, legend = ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```




```
>
```

1.3.3. Lampiran 3

Simulasi Fase $I(t)$ saat $R_0 > 1$

```
> restart :
> with(DEtools) :
> with(plots) :
> with(linalg) :
> with(student) :
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγ·X − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1280, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1280, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1290, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1290, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1300, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1300, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1340, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1340, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

$$sol1 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (13)$$

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

$$sol2 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (14)$$

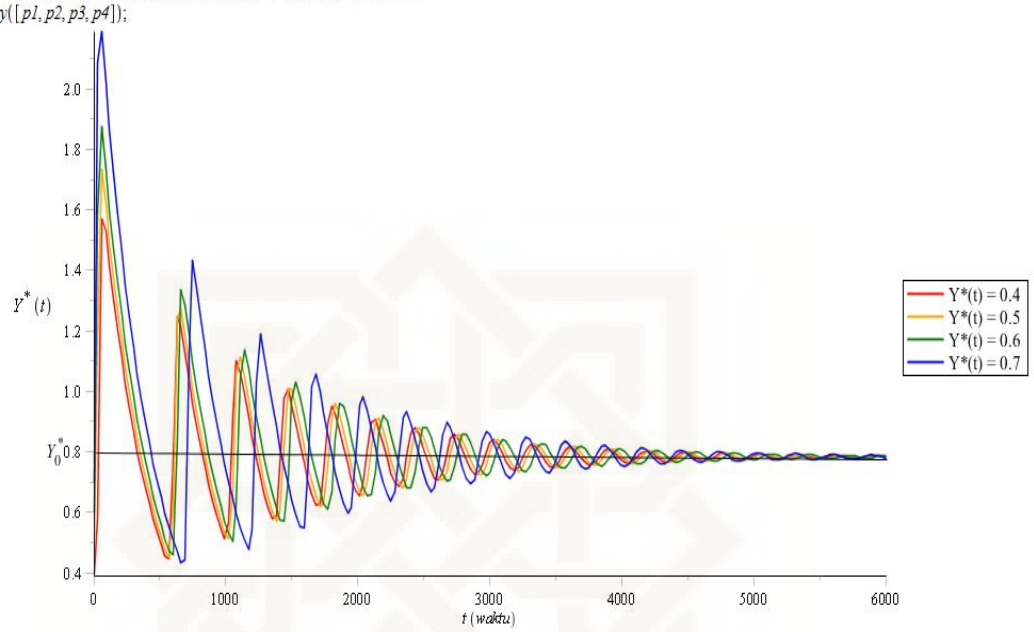
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

$$sol3 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (15)$$

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

$$sol4 := \mathbf{proc}(x\_rkf45)\ \ldots\ \mathbf{end\ proc} \quad (16)$$

```
> p1 := odeplot(sol1, [t, Y(t)], 0 ..6000, legend = ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, Y(t)], 0 ..6000, legend = ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, Y(t)], 0 ..6000, legend = ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, Y(t)], 0 ..6000, legend = ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```




```
>
```

1.3.4. Lampiran 4

Simulasi Fase $IT(t)$ saat $R_0 > 1$

```
> restart :
> with(DEtools) :
> with(plots) :
> with(linalg) :
> with(student) :
> U := [W(t), X(t), Y(t), Z(t)];
```

$$U := [W(t), X(t), Y(t), Z(t)] \quad (1)$$

```
> f := (W, X, Y, Z) → A − μ·W − δ1·Y − δ2·Z;
```

$$f := (W, X, Y, Z) \rightarrow A - \mu W - \delta_1 Y - \delta_2 Z \quad (2)$$

```
> g := (W, X, Y, Z) → β·(W − X − Y − Z)·X − μ·X − X;
```

$$g := (W, X, Y, Z) \rightarrow \beta (W - X - Y - Z) X - \mu X - X \quad (3)$$

```
> h := (W, X, Y, Z) → αγ·X − (μ + δ1)·Y;
```

$$h := (W, X, Y, Z) \rightarrow \alpha\gamma X - (\mu + \delta_1) Y \quad (4)$$

```
> i := (W, X, Y, Z) → (1 − αγ)·X − (μ + δ2)·Z;
```

$$i := (W, X, Y, Z) \rightarrow (1 - \alpha\gamma) X - (\mu + \delta_2) Z \quad (5)$$

```
> sysl := op(equate(diff(U, t), [f(op(U)), g(op(U)), h(op(U)), i(op(U))]));
```

$$sysl := \frac{d}{dt} W(t) = A - \mu W(t) - \delta_1 Y(t) - \delta_2 Z(t), \frac{d}{dt} X(t) = \beta (W(t) - X(t) - Y(t) - Z(t)) X(t) - \mu X(t) - X(t), \frac{d}{dt} Y(t) = \alpha\gamma X(t) - (\mu + \delta_1) Y(t), \frac{d}{dt} Z(t) = (1 - \alpha\gamma) X(t) - (\mu + \delta_2) Z(t) \quad (6)$$

```
> icl := W(0) = W[0], X(0) = X[0], Y(0) = Y[0], Z(0) = Z[0];
```

$$icl := W(0) = W_0, X(0) = X_0, Y(0) = Y_0, Z(0) = Z_0 \quad (7)$$

```
> equil := solve({f(w0, x0, y0, z0) = 0, g(w0, x0, y0, z0) = 0, h(w0, x0, y0, z0) = 0, i(w0, x0, y0, z0) = 0}, {w0, x0, y0, z0});
```

$$equil := \left\{w0 = \frac{A}{\mu}, x0 = 0, y0 = 0, z0 = 0\right\}, \left\{w0 \right. \quad (8)$$

$$= \frac{\mu^2 \delta_2 + \beta A \mu^2 - \delta_2 \mu^2 \alpha\gamma + \alpha\gamma \delta_1 \mu^2 + \mu \delta_2 + \delta_1 \mu \delta_2 + \beta A \delta_1 \mu + \beta A \mu + \mu \alpha\gamma \delta_1 + \beta A \mu \delta_2 - \delta_2 \mu \alpha\gamma + \beta A \delta_1 \delta_2 + \beta A \delta_1 + \delta_1 \delta_2 - \beta \alpha\gamma A \delta_1 + \delta_2 \beta \alpha\gamma A}{(1+\mu)(\mu+\delta_2)(\mu+\delta_1)\beta}, x0$$

$$= \frac{\beta A - \mu^2 - \mu}{(1+\mu)\beta}, y0 = \frac{\alpha\gamma(\beta A - \mu^2 - \mu)}{\beta(\mu + \delta_1 + \mu^2 + \delta_1 \mu)}, z0 = -\frac{(-1+\alpha\gamma)(\beta A - \mu^2 - \mu)}{(\mu+\delta_2)(1+\mu)\beta}\Bigg\}$$

> $param1 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1280, X[0] = 0.1, Y[0] = 0.2, Z[0] = 0.6\}$

$$param1 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1280, X_0 = 0.1, Y_0 = 0.2, Z_0 = 0.6, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (9)$$

>

> $param2 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1290, X[0] = 0.2, Y[0] = 0.4, Z[0] = 0.9\}$

$$param2 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1290, X_0 = 0.2, Y_0 = 0.4, Z_0 = 0.9, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (10)$$

> $param3 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1300, X[0] = 0.3, Y[0] = 0.6, Z[0] = 1\}$

$$param3 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1300, X_0 = 0.3, Y_0 = 0.6, Z_0 = 1, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (11)$$

> $param4 := \{A = 2, \mu = 0.00132, \delta_1 = 0.00139, \delta_2 = 0.00134, \beta = 0.0009, \alpha\gamma = 0.004, W[0] = 1340, X[0] = 0.4, Y[0] = 0.7, Z[0] = 1.4\}$

$$param4 := \{A = 2, \mu = 0.00132, \beta = 0.0009, \alpha\gamma = 0.004, W_0 = 1340, X_0 = 0.4, Y_0 = 0.7, Z_0 = 1.4, \delta_1 = 0.00139, \delta_2 = 0.00134\} \quad (12)$$

```
> sol1 := dsolve(eval({sysl, icl}, param1), U, type = numeric);
```

$sol1 :=$ **proc**(x_rkf45) ... **end proc** (13)

```
> sol2 := dsolve(eval({sysl, icl}, param2), U, type = numeric);
```

$sol2 :=$ **proc**(x_rkf45) ... **end proc** (14)

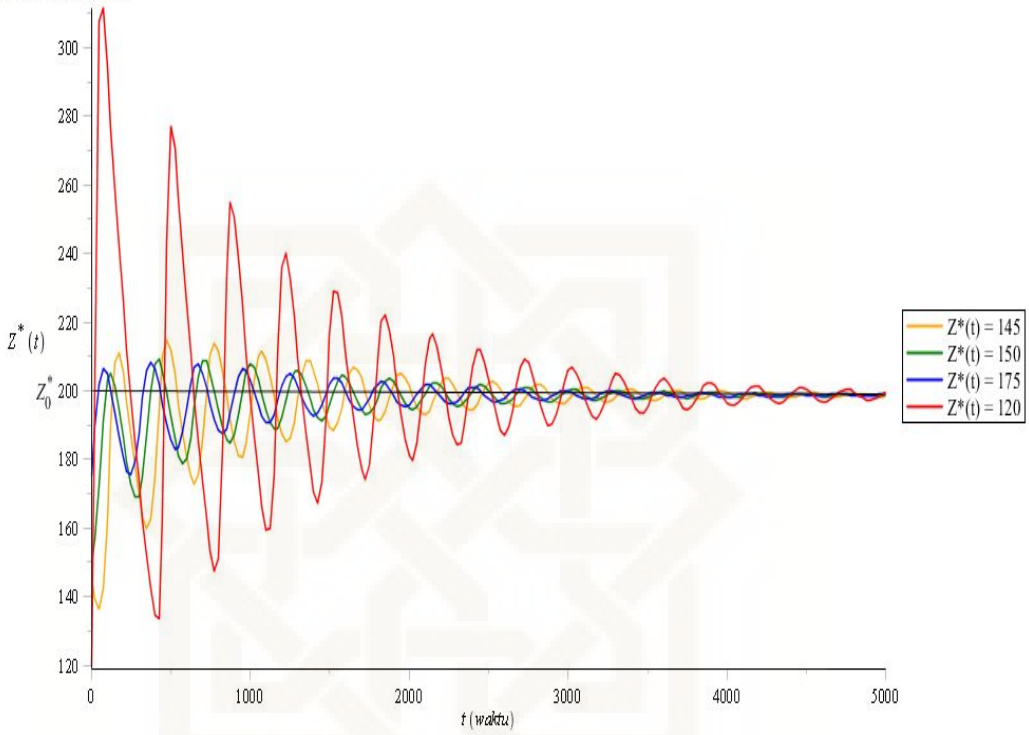
```
> sol3 := dsolve(eval({sysl, icl}, param3), U, type = numeric);
```

$sol3 :=$ **proc**(x_rkf45) ... **end proc** (15)

```
> sol4 := dsolve(eval({sysl, icl}, param4), U, type = numeric);
```

$sol4 :=$ **proc**(x_rkf45) ... **end proc** (16)

```
> p1 := odeplot(sol1, [t, Z(t)], 0 ..5000, legend= ["W", "X", "Y", "Z"]) :
> p2 := odeplot(sol2, [t, Z(t)], 0 ..5000, legend= ["W", "X", "Y", "Z"]) :
> p3 := odeplot(sol3, [t, Z(t)], 0 ..5000, legend= ["W", "X", "Y", "Z"]) :
> p4 := odeplot(sol4, [t, Z(t)], 0 ..5000, legend= ["W", "X", "Y", "Z"]) :
> display([p1, p2, p3, p4]);
```




```
>
```

RIWAYAT HIDUP

I. Identitas Diri

Nama	: Nurul Fitriyah
Jenis Kelamin	: Perempuan
Agama	: Islam
Tempat / tanggal lahir	: Magelang, 1 April 1993
Alamat	: Paten Jurang RT.01 RW.16 Rejowinangun Utara, Magelang,
Email	: pipitnurulfitriyah@gmail.com
Nama Orang tua	
Ayah	: Muhammad Najib
Ibu	: Nunung Nurjanah

II. Riwayat Pendidikan

a. 1999 – 2005	: SDN. Wates II
b. 2005 – 2008	: MTs. Pondok Pabelan
c. 2008 – 2011	: MA. Pondok Pabelan
d. 2012 – 2016	: UIN Sunan Kalijaga