ENVELOPING ALGEBRA OF A LIE ALGEBRA

A FINAL PROJECT

Submitted As a Partial Fulfillment of The Requirements
For The Degree Sarjana Sains of Mathematics

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Yogyakarta, Mei 2013

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RETNO HANA HANIFAH

Specially dedicated for my brother,

M. Azza Azizul Firda (alm),

who used to wait me back from Jogja

MOTTO

كَانَ ٱلۡبَحۡرُ مِدَادٗا لِّكَلِمَٰتِ رَبِّي لَنَفِدَ ٱلۡبَحۡرُ قَبۡلَ أَن تَنفَدَ كَلِمَٰتُ رَبِّي

وَلَوۡ جِئۡنَا بِمِثۡلِهِۦ مَدَدٗا

(QS. 18:109)

لَا يُكَلِّفُ ٱللَّهُ نَفۡسًا إِلَّا وُسۡعَهَاۚ لَهَا مَا كَسَبَتۡ وَعَلَيۡهَا مَا ٱكۡتَسَبَتۡۗ رَبَّنَا لَا تُؤَاخِذۡنَآ إِن

نَّسِينَآ أَوۡ أَخۡطَأۡنَاۚ رَبَّنَا وَلَا تَحۡمِلۡ عَلَيۡنَآ إِصۡرٗا كَمَا حَمَلۡتَهُۥ عَلَى ٱلَّذِينَ مِن قَبۡلِنَاۚ

رَبَّنَا وَلَا تُحَمِّلۡنَا مَا لَا طَاقَةَ لَنَا بِهِۦۖ وَٱعۡفُ عَنَّا وَٱغۡفِرۡ لَنَا وَٱرۡحَمۡنَآۚ أَنتَ مَوۡلَىٰنَا

فَٱنصُرۡنَا عَلَى ٱلۡقَوۡمِ ٱلۡكَٰفِرِينَ

(QS. 2:286)

إِنَّ مَعَ ٱلۡعُسۡرِ يُسۡرٗا

(QS. 94:6)

حَسۡبُنَا ٱللَّهُ وَنِعۡمَ ٱلۡوَكِيلُ

(QS. 3:173)

مَنْ لَمْ يَذُق ذُلَّ لتَّعَلُّمِ ساَعَةً تَجَرَّع ذُلَّ الْجَهْلِ طُوْلَ حَيَاتِهِ

(mahfudhot)

PREFACE

Assalamu'alaikumWr. Wb.

All praises for Allah SWT, who gives me chance, guidance and blessing to write and finish the final research project entitled *"Enveloping Algebra Of A Lie Algebra.*

This project is submitted as a partial fulfillment of the requirements for degree of Sarjana Sains (S.Si.) of Mathematics at the UIN Sunan Kalijaga of Yogyakarta. The research described herein was under the supervision of Mrs. Luluk Mauluah, M.Si. between October 2012 and June 2013. It contains the main structure, construction, properties and some important tools of universal enveloping algebra of a Lie algebra.

The research could not be finished without supports from some people. Therefore, I would like to extend my gratitude and special thanks to:

1. *Prof. Dr. H. Akh Minhaji, Ph.D,* as the dean of Faculty of Science and Technology;
2. *Muchammad Abrori, M.Kom.,* as the head of Department Mathematics, for his permission for me carrying out this project and for his much help on facing all bureaucracy and administration problems;
3. *Luluk Mauluah, M.Si.,* as my supervisor for her guidance on directing the research and solving all the problems faced as long. I also thank for her patient and appreciation of my condition;

4. My honorable parents, *H. Machfud, S.Pd.I.* and *Hj. Siti Chamidah*, also my brother *M. Sakhirul Alim, S.Pd*, and my sisters *Yuli Arsyadana Majidda* and *Intan Jauhariyah*, who keep trusting and supporting me to finish my grade;
5. *Muflihan Ahmad K*, who never gives up encourage and convince, that I can finish it best;
6. My best friends, *Rina Wanti, S.Pd.Si, Nani Septian, Aditya Saputra, S.Si, Budi Sugandi, S.Pd.Si,* and *Arif Herlambang U, S.Si,* who always belong to me on my worst time, and teach me how to be a better person;
7. *Mr. Suroto, M.Si* and *Ms. Zenith Purisha, M.Si,* as my ex-supervisors, whom with all due respect, I ask apology for giving up on last term.

This paper of final project is so far from perfect. Thus, I appreciate any constructive critics from readers. Hopefully, this paper will be worthy and useful for advanced research, either concern to Lie algebra or its universal enveloping algebra. I recognize that this is very plain and simple research which admits everyone developing and accomplishing it.

Yogyakarta, Mei 2013

Writer

Retno Hana Hanifah

TABLE CONTENTS

LIST OF SYMBOLS

$a \in R$	: a is contained on set R
$\mathcal{V} \times \mathcal{W}$	: Cartesian product of $\mathcal{V}$ and $\mathcal{W}$
$\mathbb{R}$	: Set of all real numbers
$\mathbb{Z}$	: Set of all integers
$\mathbb{Z}_n$	: Set of all integers modulo n
R	: Ring
$\mathbb{F}$	: Field
$\mathcal{V} \cup \mathcal{W}$	: Union of $\mathcal{V}$ and $\mathcal{W}$
$\mathcal{V} \subseteq \mathcal{W}$	: $\mathcal{V}$ is subset or equal to $\mathcal{W}$
$\bigcup_{i=1}^{n} \mathcal{V}_i$	: $\mathcal{V}_1 \cup \ldots \cup \mathcal{V}_n$
$M_{mxn}(\mathbb{R})$	: Set of matrices size mxn with entry $\mathbb{R}$
(a_{ij})	: Matrix
$A \Rightarrow B$	: A implies B
$A \Leftrightarrow B$	: A if and only if (iff) B
$(\Longrightarrow)$	: Sufficient condition
$(\Longrightarrow)$	: Necessary condition
$\mathcal{V} \cong \mathcal{W}$	: $\mathcal{V}$ is isomorphic to $\mathcal{W}$
$f : A \longrightarrow B$	: Map f of set A into B
$Im\ f$	: Set of image of f
$\ker f$	: Kernel of f
$\mathcal{V}/\mathcal{W}$	: Quotient of $\mathcal{V}$ by $\mathcal{W}$

$End(\mathcal{V})$	: Set of endomorphisms of $\mathcal{V}$
$[x, y]$	: Bracket of x and y (x bracket y)
$\mathfrak{gl}(\mathcal{V})$	: General linear Lie algebra
$\mathfrak{sl}_n(\mathbb{F})$	: Special linear algebra, set of matrices of size nxn whose trace 0
$\mathcal{V} \otimes \mathcal{W}$	: Tensor product of vector space (algebra) $\mathcal{V}$ and $\mathcal{W}$.
$\mathcal{V} \oplus \mathcal{W}$	: Direct sum of vector space (algebra) $\mathcal{V}$ and $\mathcal{W}$
$\mathcal{U} = \mathcal{U}(\mathcal{L})$	: Universal enveloping algebra of Lie algebra $\mathcal{L}$
$Z(\mathcal{V})$	: Center of algebra $\mathcal{V}$
$\mathcal{C}_U(\mathcal{V})$	: Centralizer of $\mathcal{V} \subset U$
$\mathcal{T}(\mathcal{V})$	: Tensor algebra of vector space $\mathcal{V}$
$\mathcal{V}^{\otimes n} = T_n(\mathcal{V})$	: $\mathcal{V} \otimes \ldots \otimes \mathcal{V}$ (n times)
$\mathcal{S}(\mathcal{L})$	: Symmetry algebra of $\mathcal{L}$
$\mathbb{Gr}(\mathcal{V})$	: Graded algebra of vector space $\mathcal{V}$
■	: End of proof

ABSTRACT

ENVELOPING ALGEBRA OF A LIE ALGEBRA

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Lie algebra is an algebra equipped with a Jacoby identity and antisymmetry property on its bilinear map. If a Lie homomorphism i, assigns the Lie algebra $\mathcal{L}$ to any unital associative algebra, $\mathcal{U}$, then the pair of $(\mathcal{U}, i)$ is called *Universal Enveloping Algebra* (UEA). An enveloping algebra satisfies the universal property and it is unique up to a unique isomorphism.

The UEA can be constructed by quotiening a tensor algebra $\mathcal{T}(\mathcal{L})$ of its two sided ideal which generated by the elements of the form $(x \otimes y - y \otimes x - [x, y])$ for $x, y \in \mathcal{L}$. The important tool to construct and describe the structure of UEA is *Poncaire-Birkhoff-Witt (PBW) theorem*. This theorem explains that a basis of Lie algebra determines the basis of UEA, so that the map i is an isomorphism. The UEA has structure of module, derivation, and center which are relative to its Lie algebra.

Focus of this paper is description of the structure of a universal enveloping algebra, including its construction and properties. It suffices with some basic concepts of abstract and linear algebra, also with structure of algebra (in general) and Lie algebra.

Keywords: Lie algebra, unital associative algebra, representation, enveloping algebra, PBW theorem.

CHAPTER I

PRELIMINARIES

A. BACKGROUNDS

Mathematics is divided into algebra, analysis, geometry, number theory, and statistics. One branch which plays big rules is *algebra*. Algebra allows us to explore the structures, relationships, and quantity. Formerly, in some millenniums, some ancient civilizations (Babylonian, Chinese, Egyptian, Indian and Greek), meant algebra as solving polynomial equations, mainly of degree four or less. It concerned to the notations, roots, and various number system, and known as *classical algebra* (Kleiner, 2007).

Algebra involved some axiomatic approaches on the early twentieth century. It is called *modern algebra* (abstract algebra). The term algebra derives from Arabic الجبر (*Al Jabr*), which literally means *reunion of broken parts*. It was taken from an Arabian mathematician, Al Khwarizmi's treatise, الكتاب المختصر في حساب الجبر والمقابلة.

One sometime only knows algebra as a part of mathematic. Moreover, algebra has its own concepts, structures, and attributes. As a structure, an algebra (over a field $\mathbb{F}$) is defined as a vector space $\mathcal{V}$ together with a bilinear map, $\mathcal{V}x\mathcal{V} \to \mathcal{V},$ which assigns $(x, y) \mapsto xy$ (Erdmann, 2006).

Mathematicians, especially algebraists, had discovered some inventions of algebra in centuries. It reached the most influential concept when *Emmy Noether* introduced and established the result of her works on her paper, "*Ideal Theory in Rings"*, on 1920s. The other important mathematician is *Sophus Lie* (1842–1899) who raise the representation theory.

Sophus Lie observed all possible group action on manifolds (in topology), he found the theory of continuous transformation group, which is called Lie group (Kleiner, 2007). Lie group is a group G which has the structure of manifold. Furthermore, the tangent space (which also considered as vector space) of Lie group at the point of identity $(T_1G = \mathcal{V})$, is identified by an exponential map, while the multiplication in G gives an operation on T_1G, that is the product of exponential maps, $\exp(x)\exp(y)$, which defined as the form $\exp\big(\mu(x,y)\big)$, for some map $\mu: \mathcal{V}x\mathcal{V} \to \mathcal{V},\ x, y \in \mathcal{V}$. As the Taylor series, this $\mu(x,y)$ is given by $x + y + \lambda(x,y) + \cdots$, for $\lambda: \mathcal{V}x\mathcal{V} \to \mathcal{V}$. Briefly, given the notation $2\lambda(x,y) = [x,y]$, which then called *commutator* (Kirrilov, 2008). Then, T_1G with the operation obtained from multiplication on G gives the structure of *Lie algebra*.

Nowadays, the Lie theory (Lie group and Lie algebra) has become a new discipline. It is useful in many parts of mathematics and physics (Humphreys, 1972). From the historical construction of a Lie algebra above, the structure of Lie algebra in algebraic views given as follows.

A ***Lie Algebra*** over field $\mathbb{F}$ is a vector space $\mathcal{V}$ over a field $\mathbb{F}$, with an operation $\mathcal{V}x\mathcal{V} \to \mathcal{V},\ (x,y) \longmapsto [x,y]$, called *bracket* or *commutator*, which is bilinear, *anti-symmetric* and satisfies the *Jacobi identity*, expressed by $\big[x,[y,z]\big] + \big[y,[z,x]\big] + \big[z,[x,y]\big] = 0$, for $x, y, z \in \mathcal{V}$. We can form a homomorphism between two Lie algebras. A linear map $\varphi: \mathcal{V}_1 \to \mathcal{V}_2$ is a homomorphism if $\varphi([x,y]) = [\varphi(x), \varphi(y)]$ for any $x, y \in \mathcal{V}_1$.

Homomorphism of a Lie algebra brings us closer to representation theory, which generally study of representing a group, vector space or module (abstract algebra) into matrices forms (linear algebra) (Hall, 2003). A *representation* of Lie algebra is a homomorphism φ of any Lie algebra $\mathcal{V}$, into a general linear algebra, $\mathfrak{gl}(\mathcal{V})$. One interesting point of representation of Lie algebra is *enveloping algebra* which appears in many application to physics, topology, and other much more fields (Kirrilov, 2008). Study of enveloping algebra reduces the study of representation of Lie algebra.

By mapping a Lie algebra $\mathcal{V}$ by φ, to a *unital associative algebra* $\mathcal{U}$ over field F, $\mathcal{U}$ is called an universal enveloping algebra of $\mathcal{V}$, if the homomorphism satisfies $\varphi([x,y]) = \varphi(x)\varphi(y) - \varphi(y)\varphi(x)$ and the universal property. The universal enveloping algebra (or UEA in short) of a Lie algebra can be constructed by quotiening a tensor algebra. The universal enveloping algebra is actually unique up to its isomorphism.

An enveloping algebra has a characteristic module, derivation, and center, which relative to the Lie algebra. The most important tool to construct and describe the structure of enveloping algebra is *Poncaire-Birkhoff-Witt (PBW) theorem*. This theorem explains that a basis of Lie algebra determines the basis of enveloping algebra.

The explorations and descriptions of enveloping algebra and its properties will be explained and discovered on this paper. So then, this paper entitled “***ENVELOPING ALGEBRA OF A LIE ALGEBRA***”.

B. LIMITATION OF THE STUDY

Since the algebra can be largely described and there are many specific types of Lie algebra, the discussion of this paper, *Enveloping Algebra of A Lie Algebra*, will be focused on describing the properties and construction enveloping algebra of arbitrary (general) Lie algebra. Although Lie algebra in its historical invention was constructed from a Lie group, we will not discover it through this topology views (Lie group).

C. FORMULATION OF THE PROBLEM

According to the backgrounds and the limitation study, then we summarize the problems will be discussed. They are:

1. What is universal enveloping algebra and how to construct it?
2. What are the properties of enveloping algebra of Lie algebra?

D. OBJECTIVES

This research is made on some purposes, i.e.:

1. To discover what is enveloping algebra and how is its construction.
2. To know the properties of enveloping algebra of a Lie Algebra.

E. SIGNIFICANCE OF THE RESEARCH

This research gives the basic knowledge of universal enveloping algebra of a Lie algebra, including the construction of enveloping algebras and some of its basic properties. Furthermore, this research especially, and the study of universal enveloping algebra in general, can be used for determining the concepts of quantum group which its representation theory can be extended to some application of group of symmetries, and construct invariant of knots and manifold.

F. LITERARY REVIEWS

The study of Lie algebra (generally Lie theory) raised in many fields, such as algebraic topology, algebraic geometry, physics and economics. Some former researches that I have them referred are described below.

Wed Giyarti (2007) on her final project has introduced the main structure of a Lie algebra, including the properties of its bracket and some examples. In the same idea, Tuti Qomariah (2010) also described Lie algebra. Even both of them provided the similar restriction on basic structures such as Lie subalgebra with its ideal and factor, also the Lie algebra of linear transformation, Tuti Qomariah drew the Lie algebra based on the definition of non-associative algebra (NAA).

James E. Humphreys on his book, (1980), defined the basic form of universal enveloping algebra and its construction by quotiening a tensor algebra by its two-sided ideal. The PBW theorem is also presented with its consequences and proofs.

Dierk Philipp Fahr (2003) on his essay, gave a detailed introduction of universal enveloping algebra. He focused on finite dimensional Lie algebra over field of characteristic zero and described the properties of universal enveloping algebra as a graded algebra.

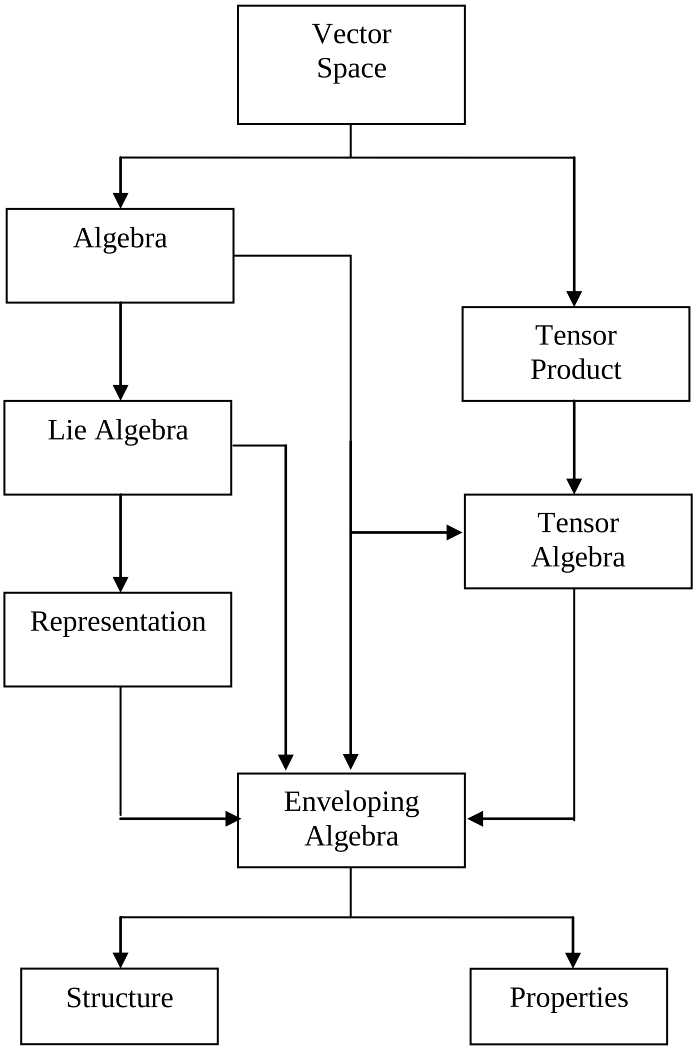
This paper will develop the introduction of Lie algebra to representation theory which has not given on both final projects above. Furthermore, it describes the structure of enveloping algebra of a Lie algebra based on Humphreys and Fahr. The properties given are not grading as given by Fahr, in order to make it easier to understand and to simplify the main idea of enveloping algebra.

G. METHODOLOGY

This research is on literary study. It allows me to collect and to explore some literatures such as journals, essays, and papers, which explained basic theory and attributes of correlated materials, such as about Lie algebra, and especially about enveloping Lie algebra.

The collected materials firstly followed by formulizing the structure and properties of algebra in general. Then, the second point is describing the concept of a Lie algebra and the main idea of representation. It also needs to describe the tensor product and tensor algebra to suffice our goal on constructing an enveloping algebra. The last step is describing the structure, including the construction and properties of universal enveloping algebra of a Lie algebra.

The following flowchart describes the processes of this research and the correspondence of each substance.



H. SYSTEMATIC SCHEME

Whole overview of this project is given below.

1. CHAPTER I

   This chapter concerns to the background of this research, also gives a whole formulation, limitation, and objectives of this research. It shows some literary reviews from former researches and describes the overview of whole project by this systematic as well.

2. CHAPTER II

   The chapter consists of theoretical backgrounds and some elementary concepts as the basic tools we must have hold, such as field, vector space, and the concept of linear transformation.

3. CHAPTER III

   The chapter describes the main concepts of algebra in general, and introduces the basic structure of Lie algebra, which underlies the structure of universal enveloping algebra, our goal. The chapter also gives the idea of representation of a Lie algebra.

4. CHAPTER IV

   Since the goal of this project, universal enveloping algebra, is also a tensorial approach, thus we briefly provide the main concept of tensor product and tensor algebra.

5. CHAPTER V

This is the aim of our research. This chapter provides the structure of universal enveloping algebra and its construction. Furthermore, it gives some properties of universal enveloping algebra, such as its uniqueness, structure of module, ideal, representation and its derivation as well. This chapter also explains some important tools, like Poncaire-Birkhoff-Witt theorem, enveloping algebra of direct sum, and center.

6. CHAPTER VI

On the last chapter, we summarize the researches done through this project, and suggest some ideas of advanced researches for the reader.

CHAPTER VI

CONCLUSIONS AND IDEAS FOR FUTURE RESEARCH

A. CONCLUSION

Given $\mathcal{U}$ as a unital associative algebra, and $\mathcal{L}$ as a Lie algebra, then $\mathcal{U}$ together with a Lie homomorphism $i: \mathcal{L} \rightarrow \mathcal{U}$, is called to be a ***Universal Enveloping Algebra***, such that it satisfies the *universal property*. Construction of an enveloping algebra can be considered by quotiening tensor algebra of a Lie algebra by its two sided ideal. The ideal of tensor algebra of a Lie algebra is on the form $\boldsymbol{x \otimes y - y \otimes x - [x, y]}$**,** for every $x, y \in \mathcal{L}$. In particular, when the Lie algebra is commutative, then its enveloping algebra corresponds to symmetry algebra.

It can also be constructed enveloping algebra of a direct sum of Lie algebra. If $\mathcal{L}_1 \oplus \mathcal{L}_2$ is a Lie algebra, and $(\mathcal{U}_1, i_1)$, $(\mathcal{U}_2, i_2)$ are respectively enveloping algebras of $\mathcal{L}_1, \mathcal{L}_2$, then the tensor product, $\boldsymbol{\mathcal{U}_1 \otimes \mathcal{U}_2}$ is the ***enveloping algebra of*** $\boldsymbol{\mathcal{L}_1 \oplus \mathcal{L}_2}$.

Poncaire-Birkhoff-Witt theorem is an important tool on describing the structure of enveloping algebra. It mainly states that symmetry algebra of a Lie algebra is isomorphic to graded algebra of its enveloping algebra. It implies that the monomial of ordered basis of Lie algebra to form the basis of its enveloping algebra and the map $i: \mathcal{L} \rightarrow \mathcal{U}$ to be injective.

Furthermore, an enveloping algebra is ***unique*** up to its unique isomorphism. It also has the structure of $\boldsymbol{\mathcal{U}}$ ***−module*** which by the representation of Lie algebra we find that it also is an $\mathcal{L}$ −module. It means that representation of enveloping algebra defines the representation of its Lie algebra. It raise the existence intertwining map of enveloping algebras**.**

The injectivity of i implies the correspondence between ideal, quotient and derivation of enveloping algebra and its Lie algebra. If I is an ideal of $\mathcal{L}$ then $\mathcal{J}$ is ideal in $\mathcal{U}$ generated by $i(I)$, then $j: x + I \to i(x) + \mathcal{J}$ is a natural homomorphism of quotient $\mathcal{L}/I$ into $\mathcal{A} = \mathcal{U}/\mathcal{J}$, and $(\boldsymbol{\mathcal{A}}, \boldsymbol{j})$ ***is a universal enveloping algebra of*** $\boldsymbol{\mathcal{L}/I}$**.** Merely, if $\mathcal{D}$ is a derivation of $\mathcal{L}$ then there will exist a unique ***derivation*** $\boldsymbol{\mathcal{D}'}$ in $\mathcal{U}$, such that $\boldsymbol{i \circ \mathcal{D} = \mathcal{D}' \circ i}$.

B. IDEAS FOR FUTURE RESEARCH

All of the results researched on previous chapters were actually lack of details substantial. Therefore, it will provide a great chance for reader to re-explore, the topic of this paper in some specific points, either about Lie algebra it self, representation theory, or the enveloping algebra.

Concerning the Lie algebra, reader can discover the structure of ideal, since it determines the type of Lie algebra; solvability, simplicity, nilpotency of reducibility of a Lie algebra. For example, the most useful type is semisimple Lie algebra, which has no nonzero solvable ideal. It can be surveyed on the derived algebra which in general define the ideal of a Lie algebra. The study of semisimple Lie algebra be explored to the root system, cartan’s criterion, radical etc.

In representation theory and universal enveloping algebra, reader can determine the concept of quantum group which defined by deforming the universal enveloping algebra of a Lie algebra. It turns applicable on theory of manifold, knot theory, physics and representation of algebraic groups. One may develop some specific points, such as *Verma module* and the *Casimir operator*, which based on the concept of center of universal enveloping algebra.

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APPENDIX

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2008	: Assistant on lecture of *Kalkulus I and II*
	Member of *Prolin (Program Olimpiade Intensif)*
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